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RESEARCH

Job-Shop Scheduling Problem with Cost Parameters for Flexible Manufacturing System

Keywords: flexible manufacturing system, cost parameters

1. INTRODUCTION

We are concerned with a problem of machine scheduling known as the general job-shop scheduling problem [1]. A flexible manufacturing system (FMS) comprises a set of n jobs J_i, $1 \le i \le n$, and a set of m machine M_k, $1 \le k \le m$. Each job (order) consists of a chain of operations, each of which needs to be processed during an uninterrupted period on a given machine. Each machine can process at most one operation at a time. If, in the course of manufacturing, one or more jobs are ready to be processed on a certain machine *and that machine is free*, a job has to be chosen from the line and passed on the machine immediately. Thus, the manufacturing process prohibits unnecessary idleness. Choosing the job from the line is carried out by using a certain decision-making rule, e.g. by undertaking a pairwise comparison [2]. If at a certain moment, more than one job is ready to be served on the machine, these jobs are compared pairwise. The winner of the first pair will be compared with the third job, etc., until only one job is left. The latter has to be chosen for the machine.

Each operation $O_{i\ell}$ is carried out under random disturbances with pregiven probability law parameters $\bar{t}_{i\ell}$ and $V_{i\ell}$. For each job (order) J_i, its due date D_i to be accomplished and delivered to the customer is pregiven (assume for simplicity that for an *already accomplished job* its delivery time to the customer equals zero). A job cannot start operating before its earliest possible moment S_i. If job J_i, $1 \le i \le n$, is accomplished later than at the due date D_i, the system pays to the customer a single cost penalty C_i^* together with an additional cost penalty C_i^{**} for each time unit of the delay. If the job is accomplished before the due date, it is not accepted by the customer until the deadline. Thus, the system is compelled to store the job until the due date and to spend C_i^{**} per time unit of storage. Note that such systems cover a broad spectrum of a job-shop FMS under random disturbances.

To simplify the problem, assume that the FMS bears neither expenses of utilizing the machines nor other working expenses (electricity, raw materials, personnel, etc.). Thus, the system has to cover only penalty and storage expenses. It can be well-recognized that, given the system's parameters D_i, C_i^*, C_i^{**} and C_i^{***}, $1 \le i \le n$, together with the initial data matrix, the total expenses depend only on values S_i which have to be determined. Optimizing the earliest possible time moments to start processing the jobs results in minimizing the total expenses. Values S_i, $1 \le i \le n$, have to be calculated beforehand and are deterministic values which deliver an optimal solution.

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2. NOTATION AND PROBLEM'S FORMULATION

To formulate the problem let us introduce the following terms:

n	-	number of jobs (orders) J_i, $1 \le i \le n$;
m	-	number of machines M_k, $1 \le k \le m$;
$O_{i\ell}$	-	ℓ-th operation of J_i $1 \le \ell \le m_i$;
m_i	-	number of operations of J_i, $m_i \le m$;
$t_{i\ell}$	-	random processing time of $O_{i\ell}$;
$\bar{t}_{i\ell}$	-	expected value of $t_{i\ell}$ (pregiven);
$V_{i\ell}$	-	variance of $t_{i\ell}$ (pregiven);
$m_{i\ell}$	-	index of the machine on which $O_{i\ell}$ is processed, $1 \le m_{i\ell} \le m$ (pregiven);
$\left\lVert \bar{t}_{i\ell},\ V_{i\ell},\ m_{i\ell} \right\rVert$	-	initial data matrix (pregiven);
S_i	-	the earliest possible time moment to start processing job J_i (to be determined);
D_i	-	due date for job J_i to be accomplished (pregiven);
$S_{i\ell}$	-	time moment job-operation $O_{i\ell}$ starts (a random value conditioned on our decisions);
p_i	-	delivery performance value of J_i, i.e., its confidence probability to be accomplished on time;
$F_{i\ell} = S_{i\ell} + t_{i\ell}$	-	the actual moment job-operation $O_{i\ell}$ is finished (a random value);
F_i	-	actual time for job J_i to be accomplished (a random value);
C_i^*	-	the penalty cost for not accomplishing job J_i on time (pregiven, to be paid once);
C_i^{**}	-	the penalty cost per *time unit* of the delay, i.e., within the period $[D_i, F_i]$ (pregiven);
C_i^{***}	-	the expenses per time unit storage in case when J_i has been accomplished before the due date (pregiven);
C_i	-	total penalty and storage expenses for job J_i;
$\bar{C} = E\left\{\sum_{i=1}^{n} C_i\right\}$	-	expected total expenses for the job-shop.

It can be well recognized that value C_i satisfies

$$C_i = \left[C_i^* + C_i^{**} \cdot (F_i - D_i)\right] \cdot \delta(J_i) + C_i^{***} \cdot (D_i - F_i) \cdot \left[1 - \delta(J_i)\right], \tag{1}$$

where

$$\delta(J_i) = \begin{cases} 1 \text{ if } F_i > D_i, \\ 0 \text{ otherwise.} \end{cases} \tag{2}$$

The problem is to determine values S_i, $1 \le i \le n$, to minimize the objective

$$\bar{C} = \underset{S_i}{\mathrm{Min}}\, E\left\{\sum_i C_i\right\} = \underset{S_i}{\mathrm{Min}}\, E\left\{\left[C_i^* + C_i^{**} \cdot (F_i - D_i)\right] \cdot \delta(J_i) + C_i^{***} \cdot (D_i - F_i) \cdot \left[1 - \delta(J_i)\right]\right] \tag{3}$$

subject to

$$S_{i1} \ge S_i,\ 1 \le i \le n. \tag{4}$$

Note that minimizing objective (3) results in the policy as follows: the management takes all measures first to accomplish jobs with higher penalty rates. These jobs have to be finished before their due dates, but as close as possible to the latter in order not to pay high storage expenses. Afterwards, the system takes similar measures for the remaining jobs. Since unnecessary idleness is prohibited, the only action is to choose suitable starting bounds S_i.

3. THE PROBLEM'S SOLUTION

We have developed the simulation model SM to simulate the FMS by implementing a decision-making rule to choose a job from the line of more than one job waiting to be processed on one and the same free machine. The rule uses cost parameters outlined above, and is based on stochastic pairwise comparison [2]. Take into account that for a job-shop manufacturing cell with pregiven initial data matrix, due dates D_i, penalty and storage rates $C_i^*, C_i^{**}, C_i^{***}$, $1 \le i \le n$, and a decision rule for choosing jobs from the line, the expected value of the job-shop total expenses $\overline{C}$ is a complicated non-linear function of values $S_1,...,S_n$. This enables solution of problem (3-4) by use of one of the coordinate descent methods [3]. We have chosen the cyclic coordinate descent method with optimized variables $S_1,...,S_n$. To solve the problem, we have developed a simulation model SM to simulate the job-shop manufacturing process together with the above outlined decision-making rule. Thus, realizing SM many times with fixed values $\{S_i\}$ enables calculation $\overline{C}(S_1,...,S_n)$. After developing SM, an initial search point $X^0 = \{S_1^{(0)},...,S_n^{(0)}$ has to be chosen, together with a constant increment $\Delta t_i > 0$, $1 \le i \le n$. First, we optimize coordinate $S_i^{(0)}$, while the other (n-1) coordinates remain unchanged. After obtaining the quasi-optimal value $S_{1\,opt}^{(0)} = S_1^{(1)}$ the latter is fixed, and the second coordinate $S_2^{(0)}$ with other unchanged coordinates $S_1^{(1)}, S_3^{(0)},...,S_n^{(0)}$, has to undergo optimization. For each coordinate $S_1^{(0)}$ value $\overline{C}(S_1,...,S_n)$ is calculated in two opposite points $\left(S_1^{(1)},...,S_{i-1}^{(1)},S_i^{(0)} - \Delta t_i, S_{i+1}^{(0)},...,S_n^{(0)}\right)$ and $\left(S_1^{(1)},...,S_{i-1}^{(1)},S_i^{(0)} + \Delta t_i, S_{i+1}^{(0)},...,S_n^{(0)}\right)$ to determine the direction of the function's decrease. The search is undertaken along those directions, i.e., values $\overline{C}\left(S_1^{(1)},...,S_{i-1}^{(1)},S_i^{(0)} + r\Delta t_i, S_{i+1}^{(0)},...,S_n^{(0)}\right)$, $r = \pm 1, \pm 2, \pm 3,...,$ are calculated. To calculate average value $\overline{C}$ at each search point, numerous simulation runs have to be undertaken to obtain representative statistics. After realizing the first iteration, i.e., determining values $S_1^{(1)}, S_2^{(1)},...,S_n^{(1)}$, we usually diminish the corresponding increments t_i, $1 \le i \le n$, and proceed to minimize $C(S_1,...,S_n)$ cyclically with respect to the coordinate variables. The algorithm terminates when the difference between two adjecent iterations $\overline{C}^{(v)}\left(S_1^{(v)},...,S_n^{(v)}\right)$ and $\overline{C}^{(v+1)}\left(S_1^{(v+1)},...,S_n^{(v+1)}\right)$ becomes less than the pregiven (prespecified) tolerance.

4. CONCLUSIONS

1. We have applied the suggested techniques for medium-size FMS successfully. Only two interations were needed to optimize the model.
2. The developed method is simple in usage and can be easily programmed on PC.

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