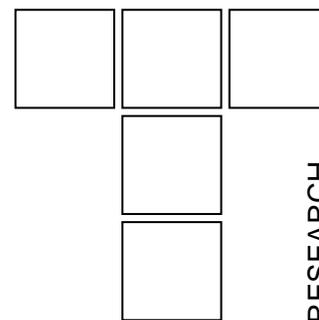*V.S. AIGBODION, S.B. HASSAN, E.T. DAUDA, R.A. MOHAMMED*

The Development of Mathematical Model for the Prediction of Ageing Behaviour for Al-Cu-Mg/Bagasse Ash Particulate Composites

The thermal ageing behaviour model of Al-Cu-Mg/Bagasse ash particulate composites with 2-10 wt% bagasse ash particles produced by double stir-casting method was developed in terms of weight fraction of bagasse ash, ageing temperature and time. Hardness values measurement was used in determining the ageing behaviour, after solution and age-hardened heat-treatment. The experimental results demonstrate that the bagasse ash was the major parameter in the ageing behaviour, followed by ageing temperature. The hardness values decreased as the ageing time increases, interaction of weight of bagasse ash, ageing time and ageing temperature. Moreover, the optimal combination of the testing parameters could be predicted. The predicted hardness values were found to lie close to that of the experimentally observed ones. The developed mathematical model can be employed for optimization of the process parameters of the ageing behaviour of Al-Cu-Mg/Bagasse ash particulate composites with respect to hardness values.

Keywords: Al-Cu-Mg alloy, Ageing temperature and time, Analysis of variance, Bagasse Ash and Linear regression

1. INTRODUCTION

The development of metal matrix composites (MMCs) is of great interest in industrial applications for lighter materials with high specific strength, stiffness and heat resistance, they form a new class of industrial materials [1]. In MMCs, aluminium-matrix composites (MMCs) reinforced with discontinuous reinforcements are very attractive because they give the best combination of strength, ductility and toughness and they can be processed by conventional methods such as casting, rolling, forging, extrusion and as a final process, machining [1-3].

Recently, there has been an increasing interest in composites containing low density and low cost reinforcements [4-5].

Among various discontinuous dispersions used bagasse ash has been found to be one of the most

inexpensive and low density reinforcement available in large quantities as solid waste from the sugar processing mill [3, 6]. Hence, composites with bagasse ash as reinforcement are likely to overcome the cost barrier for wide spread applications in automotive and small engine applications. It is therefore expected that the incorporation of bagasse ash particles in aluminium alloy will promote yet another use of this low-cost waste by-product and, at the same time, have the potential for conserving energy-intensive aluminium and thereby, reducing the cost of aluminium products [3, 6].

The age hardening characteristics of an alloy are generally modified by the introduction of reinforcement. These modifications are due to the manufacturing process, the reactivity between the reinforcement and the matrix, the size, morphology and volume fraction of the reinforcement. Strength increment due to ageing is necessary in aluminium alloys because it helps to develop acceptable mechanical properties [1].

The earlier works [7-11] concluded that the addition of discontinuous ceramic particles into aluminium matrix resulted in the dislocation

V.S. Aigbodion, S.B. Hassan, E.T. Dauda, R.A. Mohammed
Department of Metallurgical and Materials Engineering, Ahmadu Bello University, Samaru, Zaria, Nigeria.
Email: aigbodionv@yahoo.com

generation leading to different ageing kinetics compared to monolithic alloys. Thus, there is a complexity involved in the ageing process of composites when compared with that of unreinforced alloys [8]. Hence, an attempt has been made in the present investigation to study and systematically record the effects of heat-treatment parameters on hardness of Al-Cu-Mg/BAp composite. The age - hardening model of these composites was developed based on the weight fraction of bagasse ash, ageing temperature and time. Further more analysis of variance (ANOVA) is employed to investigate the testing characteristics of the Al-Cu-Mg/BAp composite**s.**

2. MATERIALS AND EXPERIMENTAL PROCEDURE

The Bagasse ash used in these study were characterized and the results shown in Table 1. Composites used in this study were A2009Al–Bagasse ash particles composites containing 2-10 wt% Bagasse ash particles. The samples were produced using the double stir casting method [1, 3] by keeping the percentage of copper and magnesium constant (3.7%Cu and 1.4%Mg) according to the recommended standard to produced alloy of type A2009 [3, 6] with 2-10Wt% Bagasse ash of particles size of 64μm. A control sample without the Bagasse ash was also produced with this method. After casting, the specimen was machined into hardness coupons for the purpose of determining the thermal ageing behaviour of the produced composites.

Table 1. Composition and properties of bagasse ash particles

Constituent Formula	Cliftonite (C), Quartz (SiO_2), Moissanite (SiC), Titanium Oxide (Ti6O)
Density and phase	1.95g/cm^3 and Solid
Refractoriness	1600°C
Appearance(color)	Black-Odorless powder
Size	64μm
Hardness values	75.05 HRB

The test coupons were polished at both the ends. The test samples were solution heat-treated at temperature of 500°C in an electrically heated furnace, soaked for 3 hours at this temperature and then rapidly quenched in warm water at 65°C. Thermal ageing of the quenched samples were carried out at temperatures of 100, 200 and 300°C, for various ageing times until the peak ageing is exceeded [3]. The ageing characteristic of these

grades of composites was evaluated using hardness values obtained from age-hardening samples.

The test coupons were polished at both the ends. The test samples were solution heat-treated at temperature of 500°C in an electrically heated furnace, soaked for 3 hours at this temperature and then rapidly quenched in warm water at 65°C. Thermal ageing of the quenched samples were carried out at temperatures of 100, 200 and 300°C, for various ageing times until the peak ageing is exceeded [3]. The ageing characteristic of these grades of composites was evaluated using hardness values obtained from age-hardening samples.

The sequence of operations involved in the heat treatment is solutionizing, quenching, thermal ageing and air-cooling. In the above mentioned sequence, the independently controllable predominant process parameters considered for the investigation are ageing temperature (*AT*), Bagasse ash particles (BAp) and ageing time (*At*) at two levels [9]. The two levels decided for each of the three process parameters with their units and notations are given in Table 2.

Table 2. Statistical design of the ageing process

Factors	Low level	High level
Temperature (A_T)	100°C	300°C
% wt of (BAp)	2.0	10.0
Time(A_t)	1hr	14hrs

Full factorial design is a statistical tool to analyze a set of results with minimum number of experiments. The methods of designing such experiments are dealt with in literature [12]. In the present study, due to narrow range of the process parameters chosen, it was decided to use a two level full factorial design. The eight sets of coded conditions of experiments based on 2^3 full factorial designs are given in Table 3 [12].

Table 3. Factorial design of the ageing process showing treatment combination

Exp.number	Temperature level	% wt of BAp level	Time level
1	-1	-1	-1
A_T	+1	-1	-1
BAp	-1	+1	-1
A_T BAp	+1	+1	-1
A_t	-1	-1	+1
A_T A_t	+1	-1	+1
BAp A_t	-1	+1	+1
A_T BAp A_t	+1	+1	+1

Coded= -1(low level), +1(upper level),
BAp (Bagasse ash particles)

The test results were recorded against the standard order of sequence as shown in Table 4. The sum of squares for main and interaction effects was calculated using Yates algorithm. The significant factors (main and interaction) were identified by analysis of variance (ANOVA) technique as shown in Table 5 [13].

Table 4. Standard Order of Test Sequence and Result

Exp. number	Temperature level (^{o}C)	% wt of BAp level	Time level (hours)	Hardness values (HRB)
1	100	2	1	36.2
A_T	300	2	1	40.5
BAp	100	10	1	51.0
A_T BAp	300	10	1	54.0
A_t	100	2	14	44.0
$A_T A_t$	300	2	14	40.7
BAp A_t	100	10	14	53.8
A_T BAp A_t	300	10	14	50.0

Table 5. Analysis of Variance table to identify significant factors influencing hardness

Factors	Mean effect	Sum of squares (SS)	Degree of freedom	Mean Square (M_s)	$F_{Cal} = \dfrac{M_S}{Error M_S}$
Main effect					
A_T	10.40	108.50	1	108.50	13.76
BAp	11.85	140.43	1	140.43	17.80
A_t	-1.70	2.89	1	2.89	0.37
Interactions					
A_TBAp	1.95	3.80	1	3.80	0.48
$A_T A_t$	-2.30	5.29	1	5.29	0.67
BApA_t	-0.20	0.04	1	0.04	0.05
$A_T A_t$BAp	-1.20	1.45	1	1.45	0.18
Error	7.75	63.12	8	7.89	

3. DEVELOPMENT OF MATHEMATICAL MODEL

The model for the age-hardening behvaiour of these composites was obtained by representing the hardness values by W, the response function can be expressed by equation below:

$$W = f(A_T, B_P, A_t) \tag{1}$$

Where:
A_T = Ageing temperature
B_P = weight % bagasse ash particle
A_t = Ageing time

The model selected includes the effects of main variables first-order and second-order interactions of all variables. Hence the general model is written as:
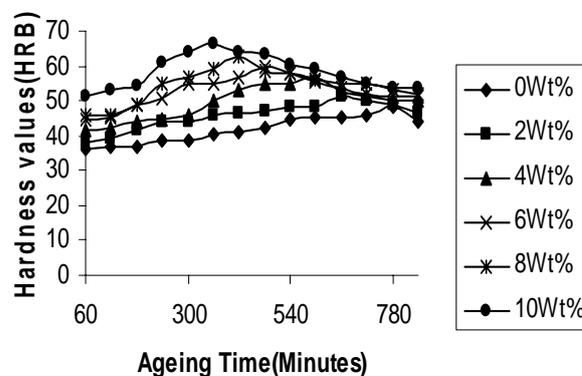
$$W = \beta_o + \beta_1 A_T + \beta_2 B_P + \beta_3 A_t + \beta_4 A_T B_P + \beta_5 A_T A_t + \beta_6 B_P A_t + \beta_7 A_T B_P A_t \tag{2}$$

Were β_o is average response of W and $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7$ are coefficients associated with each variable A_T, B_P, A_t and interaction are calculated using the linear regression method [13].
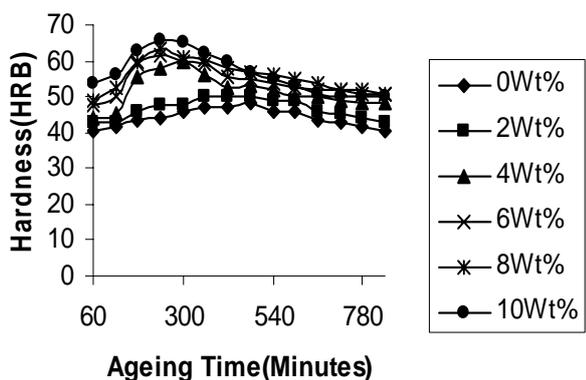
4. RESULTS AND DISCUSSION

The results obtained for the ageing behaviour of these composites are presented in Figures 1-3.

Figure 1. Variation of the Hardness values with Ageing Time at Ageing Temperature of 100°C
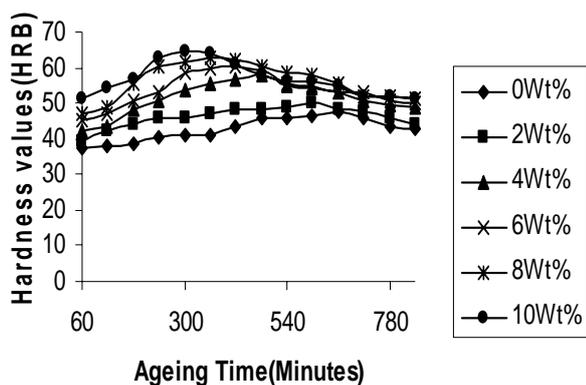
Figure 2. Variation of the Hardness values with Ageing Time at Ageing Temperature of 200°C

Figure 3. Variation of the Hardness values with Ageing Time at Ageing Temperature of 300°C

From Figures 1-3, it can be seen that there is steep rise in hardness values of each grade of the composite at initial stages for all ageing temperatures and then fell after reaching the various peak ageing time, corresponding to over ageing. However, at higher ageing temperature the materials developed peak hardness at shorter ageing time, because the rate of precipitation of the second phase materials is faster and hence increases in hardness values. The time to obtain peak hardness is shorter according to the sequence: 100°C > 200°C > 300°C (see Figures 1-3).

The thermal age hardening behavior of the Al-Cu-Mg/BAp particulates composites are similar to Al-Cu/SiC particulates as reported by Suresh et al [9] i.e. hardness continuously increases to a maximum during thermal ageing and them decreases later due to over ageing. It is interesting to note that in the reinforced aluminium alloy metal-matrix, as the volume fraction of bagasse ash particle increase to 10 wt% in the aluminium alloy, there is a monotonic reduction in the time required to reach peak hardness (see Figure 3).

The 10 Wt% BAp addition yielded the highest hardness value. As far as hardening behavior of the composites is concerned, particle addition in the matrix alloy increases the strain energy in the periphery of the particles in the matrix and these tendencies may be due to the formation of the dislocation at the boundary of the ceramic particles by the difference in the thermo-expansion coefficient between the matrix and ceramic particles during solution treatment and quenching since a lot of dislocations generate in the main matrix/particle interface [9-10]. Thus, dislocations cause the hardness increase in composite as well as residual stress increase because of acting as non-uniform nucleation sites in the interface following the age treatment. It is thought that the higher the amount of the ceramic particles in the matrix, the higher the density of the dislocation, and as a result, the higher the hardness of the composite [8, 11].

From the result of factorial design (see Table 5), the Wt% of BAp appears to be the most important variable with main effect of 11.85HRB followed by temperature with 10.48HRB and time -1.70HRB. The analysis shows that raising the temperature from 100 to 300°C would result in increase in hardness values by 10.40HRB, while allowing ageing to continue from 1 to 14hours would result in decreased hardness values by 1.7HRB and increasing the Wt% of BAp from 2 to 10 would result in increasing the hardness values by 11.85HRB.

The estimated interactions between temperature and Wt% of BAp; temperature and time; Wt% of BAp and time and then between the three factors, temperature, Wt% of BAp and time are 1.95HRB, -2.30HRB, -0.20HRB and -1.20HRB respectively. This means that raising temperature and time would result in decrease in hardness values by 2.30HRB whereas raising all the three factors would result in decrease in hardness values by 1.2HRB. The main reason for the decreased in hardness values when time increase from 1 to 14 hours is due to the fact that after peak ageing time has been reached further ageing will not lead to any increases in hardness as a result of over- ageing [8], this facts can be evident from the figures 1-3.

The values of *F*calculated (*F*=Fishers distribution) are compared with *F*critical. *F* distribution critical values for {1,8} degrees of freedom are above 5.32 for 95% and above 11.26 for 99% confidence level [13]. Thus, from Table 5, it can be observed that only bagasse ash and temperature have significant factor on hardness values, all others factor have no

significant effect. From this statistical analysis, %Wt of BAp and temperature is the most important parameter in the ageing behaviour of Al-Cu-Mg/BAp particulate composites.

The model equation was obtained after calculating each of the coefficients of Eq. 2. The developed model equation for the ageing behaviour of the composites can be expressed as:

$$W = 48.55 + 3.63A_T + 4.57B_P - 1.42A_t - 0.23A_TB_P - 1.8A_TA_t - 3.45B_PA_t - 0.1A_TB_PA_t \qquad (3)$$

Substituting the coded values of the variables for any experimental condition in Eq. 3, the hardness values for the ageing behaviour of the composites can be calculated. Table 6 and Figure 4 shows the predicted values along with the actual experimental values in different experimental conditions.

Table 6. Comparison of the actual with the predicted result

Exp.number	Temperature level	% wt of BAp level	Time level	Hardness values(HRB)	
				Actual	Predicted
S1	0	1	1	50.00	48.24
S2	+1	0	+1	50.70	48.76
S3	+1	+1	0	60.00	56.50
S4	0	−1	+1	44.30	46.00
S5	1	0	−1	53.90	55.40
S6	0	0	+1	44.50	45.30

Coded= −1 (low level), +1 (upper level), 0 (Base line), BAp (Bagasse ash particles)
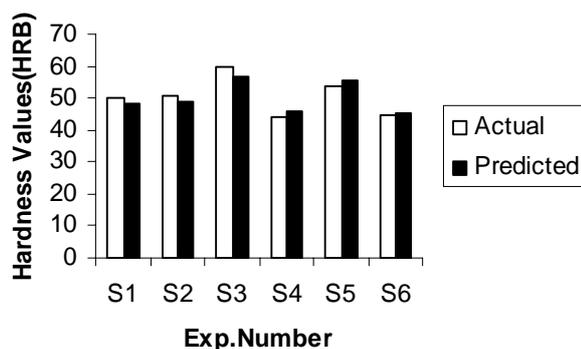
Figure 4. Variation of the Hardness values with Experimental Numbers

It is evident from Table 5 and Figure 4 that the actual experimental values are in close proximity with the predicted values. These facts suggested reasonably good reliability of the equation to predict the ageing behaviour of the Al-Cu-Mg/BAp composites within the selected experimental domains.

The adequacy of the developed model was checked by determining the correlation coefficient R of the result in Table 6. Since the values of R in literature lies in the ranges of -1 to +1 and +1 means perfect relationship [13]. Hence, the calculated value of R obtained in this work is 0.94, which means that the developed model has high correlation with the experimental values. The trend model equation substantiates conclusion that bagasse ash and temperature has significant effect on hardness values.

5. CONCLUSIONS

Hardness tests were performed on smooth samples of Al-Cu-Mg/BAp composites subjected to solution heat treatment and ageing schedule. The individual and interaction effects of the parameters, viz ageing temperature, bagasse ash and aging time were studied. The conclusions derived from this study are as follows:

1. At higher ageing temperature the composites developed peak hardness at shorter ageing time.

2. As the volume fraction of bagasse ash particle increase to 10Wt% in the aluminium alloy there is a monotonic reduction in the time required to reach peak hardness

3. The main and the interaction effects of significant combination of heat-treatment parameters within the range of investigation of Al-Cu-Mg/BAp composite can be studied emphatically by factorial experimentation technique.

4. The developed mathematical model can be used to predict the hardness values in terms of heat-treatment process parameters obtained from any combinations within the ranges studied.

5. The bagasse ash and temperature has the maximum influence on hardness values.

6. The results obtained from the statistical analysis are in good agreement with the experimental findings for the bagasse ash, ageing time and ageing temperatures. It was found that hardness increases with increasing weight fraction of bagasse ash in the alloy and decreases with increasing ageing time.

7. The developed mathematical model can be employed for optimization of the process parameters of Al-Cu-Mg/BAp composites with respect to hardness values.

REFERENCES

[1] V. S. Aigbodion and S. B. Hassan: *Effect of Thermal Ageing Characteristics of Al-Si-Fe/SiC Particulate Composite Synthesized By Double Stir Casting*, The Journal of Engineering Research(TJER), Vol. 7, No. 1, (2010) 53-61.

[2] M. Sudarshan, K Surappa: *Synthesis of fly ash particle reinforced A356 Al composites and their characterization*, Materials Science and Engineering A, 480 (2008) 117–124.

[3] V.S. Aigbodion: *Potential of using Bagasse ash particle in Metal Matrix Composite*, PhD Work, On-going, Department of Metallurgical and Materials Engineering, Ahmadu Bello University, Samaru, Zaria, Nigeria (2008).

[4] J. Bienia, M. Walczak, B. Surowska., J. Sobczaka: *Microstructure and Corrosion Behaviour of Aluminum Fly Ash Composites*, Journal of Optoelectronics and Advanced Materials Vol. 5, No. 2 (2003) 493 – 502.

[5] V. Kevorkijan, B. Utar, M. Torkar, G. Hiarmetta: *Preparation of Al Based Composite Reinforced With Fine Dross Particles*, ISSN 1318-0010 KZLTET (1998) 32 (6) 539.

[6] S. B. Hassan and V. S. Aigbodion**:** *The study of the Microstructure and Interfacial Reaction of Al-Cu-Mg/Bagasse Ash Particulate Composite*, Journal of Alloy & Compounds, 491 (2010) 571–574.

[7] S. Valdez, B. Campillo, R. Pérez, L. Martínez, H. García: *Synthesis and microstructural characterization of Al–Mg alloy–SiC particle composite*, Materials Letters 62 (2008) 2623–2625.

[8] S.B. Hassan, O. Aponbiede and V.S. Aigbodion: *Precipitation hardening characteristics of Al- Si-Fe/SiC particulate composites*, Journal of alloys & compounds 466 (2008), 268-272

[9] S. Suresh, T. Christman, Y. Sugimura: *Accelerated aging in cast Al alloy—SiC particulate Composite*, Scr. Metall. 23 (1989) 1599–1602.

[10] P. Appendino, C. Badini, F. Marino, A. Tomasi: *6061 Aluminium alloy—SiC particulate Composite: a comparison between aging behaviour in T4 and T6 treatments*, J. Mater. Sci. Eng. A.135 (1991) 275–279.

[11] L. Salvo, M. Svery: *Effect of reinforcement on age-hardening of cast 6061-AlSiC and 6061 Al-Al2O3 particulate composite*, Metall. Trans. A 22A (1996) 2553–2653.

[12] Y. Sahin.: *The prediction of wear resistance model for the metal matrix composites*, Wear 258 (2005) 1717–1722.

[13] I. Miller, J.E. Freund: *Probability and Statistics for Engineers*, Prentice Hall India Ltd., India (2001), 125-140.