

## On the Performance of Pivoted Curved Slider Bearings: Rabinowitsch Fluid Model

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### ABSTRACT

The present theoretical analysis is to investigate the effect of non-Newtonian Pseudoplastic & Dilatant lubricants (lubricant blended with viscosity index improver)–Rabinowitsch fluid model on the dynamic stiffness and damping characteristics of pivoted curved slider bearings. The modified Reynolds equation has been obtained for steady and damping states of the bearing. To analyze the steady state characteristics and dynamic characteristics, small perturbation theory has been adopted. The results for the steady state bearing performance characteristics (steady state film pressure, load carrying capacity and centre of pressure) as well as dynamic stiffness and damping characteristics have been calculated numerically for various values of viscosity index improver using Mathematical 7.0 and it is concluded that these characteristics vary significantly with the non-Newtonian behavior of the fluid consistent with the real nature of the problem.

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### 1. INTRODUCTION

In recent years, tribologists have done a great deal of work to increase the efficiency of stabilizing properties of non-Newtonian lubricants by addition of small amounts of long chain polymer solutions such as Polyisobutylene, Ethylene propylene etc. The use of additives minimizes the sensitivity of the lubricant to the change in the shearing strain rate. Further, the viscosity of these lubricants exhibits a non-linear relationship between the shearing stress and shearing strain rate.

In last few decades, the rheological effects of non-Newtonian lubricants based on different fluid models like Power Law and Couple Stress fluid model have been studied for the performance characteristics of Journal, Squeeze film, Annular disks and Externally Pressurized bearings. To study the performance properties of bearings lubricated with non-Newtonian lubricants, Rabinowitsch fluid model is one of the fluid models to establish the non-linear relationship between the shearing stress and shearing strain rate.

In the Rabinowitsch Fluid Model, the following empirical stress-strain relation holds for one dimensional fluid flow:

$$\tau_{xy} + \kappa \tau_{xy}^3 = \mu \frac{\partial u}{\partial y} \quad (1)$$

where  $\mu$  is the zero shear rate viscosity,  $k$  is the non-linear factor responsible for the non-Newtonian effects of the fluid which will be referred to as coefficient of Pseudoplasticity in this paper. This model can be applied to Newtonian lubricants for  $k = 0$ , Dilatant lubricants for  $k < 0$ , and Pseudoplastic lubricants for  $k > 0$ . The experimental analysis of this model for the lubricants for Journal bearing has been justified by Wada and Hayashi [1] indicating the film pressure and load capacity for these lubricants is smaller than those of the Newtonian fluids. Afterwards, the theoretical study of bearing performance with non-Newtonian lubricants using this and other models were done by Bourging and Gay [2] on Journal bearing, Hsu and Saibel [3], Hashimoto and Wada [4] on circular plates bearing, Usha and Vimla [5] on Squeeze film between two plane annuli and Hung [6] on infinitely wide parallel rectangular plates. The dynamic analysis of the slider and other bearings has also been the centre of attention of various researchers in recent decades. Sharma and Pandey [7] presented the dynamic analysis of bearings. Some other appreciable contributions to bearings lubrication is by Shimpi et al. [8], Srikanth et al. [9] and Shenoy et al. [10] on slider, hydrostatic thrust and journal bearings. However, none of the investigators have put up their attention to study theoretically, the problem of isothermal, incompressible laminar flow lubricant for pivoted curved slider bearings taking into account the Rabinowitsch fluid model.

In the present paper, the effect of non-Newtonian lubricants on the steady and dynamic characteristics of pivoted curved slider bearing has been investigated using Rabinowitsch Fluid Model. Since, the problem is of non-linear nature in its theoretical investigation, the numerical results for steady state pressure, load capacity, centre of pressure, dynamic stiffness and damping coefficients have been obtained using Mathematica 7.0.

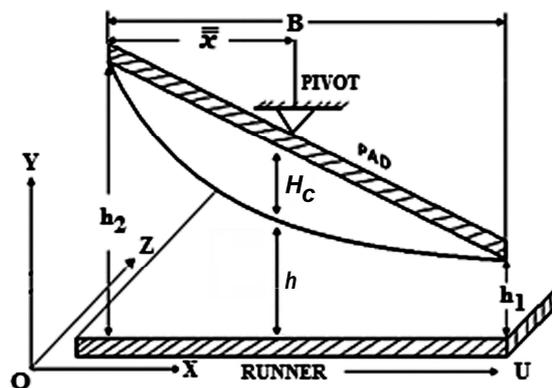


Fig. 1. Schematic diagram of pivoted curved slider bearing.

## 2. CONSTITUTIVE EQUATIONS AND BOUNDARY CONDITIONS

The physical configuration of a curved slider bearing is shown in Fig. 1. The bearing is consisting of two surfaces, a plane and a curved slider, separated by a lubricant film. The plane is moving with a uniform velocity  $U$ , as shown in the Fig. 1, while the curved surface is at rest. The lubricant in the system is taken as non-Newtonian Rabinowitsch fluid. The body forces and body couples are assumed to be absent.

Under the assumptions of hydrodynamic lubrication applicable to thin film as considered by Dowson [11], the field equations governing the one dimensional motion of an incompressible non-Newtonian fluid-Rabinowitsch fluid model used by Wada and Hayashi [1] in cylindrical polar co-ordinate system are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xy}}{\partial y} \quad (3)$$

$$\frac{\partial p}{\partial y} = 0 \quad (4)$$

which are solved under the following boundary conditions :

$$u = U, \quad v = 0 \quad \text{at } y = 0 \quad (5)$$

$$u = 0 \quad \text{at } y = h \quad (6)$$

$$v = V = -\frac{\partial h}{\partial t} \quad \text{at } y = h \quad (7)$$

$$p = 0 \quad \text{at } x = 0, B \quad (8)$$

where  $u$  and  $v$  are the velocity components in  $x$  and  $y$  directions and  $h$  is the film thickness between the bearings plates respectively.

### 3. ANALYSIS

Integration of equation (3) with respect to  $y$  gives:

$$\tau_{xy} = \frac{\partial p}{\partial x} y + c_1 \quad (9)$$

From equation [9] and [1] we get:

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \left[ \left( \frac{\partial p}{\partial x} y + c_1 \right) + \kappa \left( \frac{\partial p}{\partial x} y + c_1 \right)^3 \right] \quad (10)$$

Integrating equation (10) under the boundary conditions (5,6), we get:

$$u = \frac{1}{\mu} \left[ \frac{1}{2} \frac{\partial p}{\partial x} y(y-h) + \kappa \left\{ \frac{1}{4} y^4 - \frac{1}{2} h y^3 + \frac{3}{8} h^2 y^2 - \frac{1}{8} h^3 y \right\} \right] + U \left[ 1 - \frac{y + \kappa \left( \frac{\partial p}{\partial x} \right)^2 \left( y^3 - \frac{3}{2} h y^2 + \frac{3}{4} h^2 y \right)}{h \left( 1 - \frac{1}{4} \kappa \left( \frac{\partial p}{\partial x} \right)^2 h^2 \right)} \right] \quad (11)$$

Integrating the equation of continuity (2) under the relevant boundary conditions (5,7) for  $v$  using (11), the modified Reynolds equation is obtained as:

$$\frac{\partial}{\partial x} \left[ h^3 \frac{\partial p}{\partial x} + \frac{3}{20} \kappa h^5 \left( \frac{\partial p}{\partial x} \right)^3 \right] = -6\mu U \frac{\partial h}{\partial x} + 12\mu \frac{\partial h}{\partial t} \quad (12)$$

The expression for the film thickness in dynamic condition is given as:

$$h \equiv h(x, t) = h_s(x) + h_m(t) \quad (13)$$

where  $h_s(x)$  is the steady state film thickness:

$$h_s(x) = H_c \left[ 4 \left( \frac{x}{B} - \frac{1}{2} \right)^2 - 1 \right] + h_1 \left[ 1 + (r_b - 1) \left( 1 - \frac{x}{B} \right) \right] \quad (14)$$

given by Abramovitz [12] and used by Kapur [13], and  $h_m(t)$  is the variation of minimum film thickness with the time in dynamic condition which becomes zero in steady state, where  $r_b = h_2/h_1$  and  $H_c$  is the height of crown segment

(i.e. the maximum height of the curved segment) for film shape.

Introducing the dimensionless parameters:

$$\Delta = \frac{H_c}{h_1} \quad \text{and} \quad \bar{x} = \frac{x}{B},$$

the dimensionless Reynolds equation becomes:

$$\frac{\partial}{\partial \bar{x}} \left[ \bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{3}{20} \alpha \bar{h}^5 \left( \frac{\partial \bar{p}}{\partial \bar{x}} \right)^3 \right] = 6 \frac{\partial \bar{h}_s}{\partial \bar{x}} + 12 \vartheta \frac{\partial \bar{h}_m}{\partial \tau} \quad (15)$$

where

$$\bar{h}_s = \Delta \left[ 4 \left( \bar{x} - \frac{1}{2} \right)^2 - 1 \right] + \left[ 1 + (r_b - 1)(1 - \bar{x}) \right] \quad (16)$$

And  $v = B\omega/U$  is the damping parameter,  $\alpha = \kappa \mu^2 U^2 / h_1^2$  is the parameter of Pseudoplasticity responsible for the non-Newtonian behaviour of the lubricant. For  $\alpha = 0$  equation (15) becomes the classical Reynolds equation for slider bearing with Newtonian lubricant.

For the present problem,  $\bar{h}_m$  and the pressure under damping condition is of the form:

$$\bar{h}_m = \varepsilon e^{i\tau} \quad (17)$$

$$\bar{p} = \bar{p}_o + \varepsilon \bar{p}_1 e^{i\tau} \quad (18)$$

where  $\varepsilon$  is dimensionless amplitude of oscillation[14] (i.e. maximum variation of film thickness due to small oscillation) and  $\bar{p}_o$  is the steady state pressure.

The Reynolds equations under steady state become:

$$\frac{d}{d\bar{x}} \left[ \bar{h}_s^3 \frac{d\bar{p}_o}{d\bar{x}} + \frac{3}{20} \alpha \bar{h}_s^5 \left( \frac{d\bar{p}_o}{d\bar{x}} \right)^3 \right] = 6 \frac{d\bar{h}_s}{d\bar{x}} \quad (19)$$

and the Reynolds equation under damping condition becomes:

$$\frac{d}{d\bar{x}} \left[ \bar{h}_s^3 \left\{ 1 + \frac{9}{20} \alpha \bar{h}_s^2 \left( \frac{d\bar{p}_o}{d\bar{x}} \right)^2 \right\} \frac{d\bar{p}_1}{d\bar{x}} \right] = 12 \vartheta i \quad (20)$$

$$- 3 \frac{d}{d\bar{x}} \left[ \bar{h}_s^2 \left\{ \frac{d\bar{p}_o}{d\bar{x}} + \frac{5}{20} \alpha \bar{h}_s^2 \left( \frac{d\bar{p}_o}{d\bar{x}} \right)^3 \right\} \right]$$

**4. STEADY STATE PRESSURE**

Integrating equation (20) under the condition:

$$\bar{p}_o = 0 \quad \text{at} \quad \bar{x} = 0, 1$$

using small perturbation technique, the dimensionless steady state film pressure becomes

$$\bar{p}_o(\bar{x}) = \int_0^{\bar{x}} \frac{C_{oo} + 6\bar{h}_s}{\bar{h}_s^3} d\bar{x} + \alpha \left[ C_{o1} \int_0^{\bar{x}} \frac{1}{\bar{h}_s^3} d\bar{x} - \frac{3}{20} \int_0^{\bar{x}} \frac{(C_{oo} + 6\bar{h}_s)^3}{\bar{h}_s^7} d\bar{x} \right] \quad (21)$$

where

$$C_{oo} = -6 \int_0^1 \frac{1}{\bar{h}_s^2} d\bar{x} / \int_0^1 \frac{1}{\bar{h}_s^3} d\bar{x} \quad (22)$$

and

$$C_{o1} = \frac{3}{20} \int_0^1 \frac{(C_{oo} + 6\bar{h}_s)^3}{\bar{h}_s^7} d\bar{x} / \int_0^1 \frac{1}{\bar{h}_s^3} d\bar{x} \quad (23)$$

**5. STEADY STATE LOAD CAPACITY**

The dimensionless load carrying capacity of the bearing can be calculated as:

$$\bar{W} = \int_0^1 \bar{p}_o d\bar{x} \quad (24)$$

In order to avoid very lengthy procedure of integration, numerical integration method (Gaussian Quadrature formula) has been adopted to obtain the numerical values of the load capacity using Mathematica 7.0. The Gaussian Quadrature formula has been adopted due to its higher rate of convergence in comparison with the other numerical methods like Trapezium Rule, Midpoint Rule and Simpson's one third and three eighth formulae.

**6. DYNAMIC STIFFNESS AND DAMPING CHARACTERISTICS**

In order to obtain the analytical solution of the dynamic Stiffness Coefficient  $\bar{S}_D$  and dynamic Damping Coefficient  $\bar{C}_D$ , the perturbed film pressure gradient  $d\bar{p}_1/d\bar{x}$  is obtained from

equations (20) and (21) which is given as follows:

$$\frac{d\bar{p}_1}{d\bar{x}} = \frac{C_1 + 12\bar{h}_s \bar{x} - 3\bar{h}_s^2 \left[ f(\bar{x}) + \frac{5}{20} \alpha \bar{h}_s^2 f(\bar{x})^3 \right]}{\bar{h}_s^3 \left[ 1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2 \right]} \quad (25)$$

Further, the perturbed pressure:

$$\bar{p}_1(\bar{x}) = \bar{p}_{11}(\bar{x}) + i\bar{p}_{12}(\bar{x}) \quad (26)$$

is obtained on integrating the equation (26) under the boundary conditions:

$$\bar{p}_1 = 0 \quad \text{at} \quad \bar{x} = 0, 1$$

where

$$f(\bar{x}) = \frac{d\bar{p}_o}{d\bar{x}},$$

$$\bar{p}_{11}(\bar{x}) = C_{11} \int_0^{\bar{x}} \left[ \frac{1}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x} - 3 \int_0^{\bar{x}} \left[ \frac{f(\bar{x}) \{1 + \frac{5}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}}{\bar{h}_s \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x} \quad (27)$$

$$\bar{p}_{12}(\bar{x}) = C_{12} \int_0^{\bar{x}} \left[ \frac{12}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x} + \int_0^{\bar{x}} \left[ \frac{12\bar{x}}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x} \quad (28)$$

with

$$C_{11} = \frac{3 \int_0^1 \left[ \frac{f(\bar{x}) \{1 + \frac{5}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}}{\bar{h}_s \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x}}{\int_0^1 \left[ \frac{1}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x}}$$

and

$$C_{12} = - \frac{\int_0^1 \left[ \frac{12\bar{x}}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x}}{\int_0^1 \left[ \frac{1}{\bar{h}_s^3 \{1 + \frac{9}{20} \alpha \bar{h}_s^2 f(\bar{x})^2\}} \right] d\bar{x}}$$

The film force  $F_D$ , under the damping condition is given by:

$$F_D = L \int_0^B p_1(x) dx \quad (29)$$

In the dimensionless form:

$$\bar{F}_D = \int_0^1 \bar{p}(\bar{x}) d\bar{x} \quad (30)$$

The resulting dynamic force can be expressed in the terms of linearized damping and stiffness coefficient<sup>14)</sup> as follows:

$$F_D \varepsilon e^{i\tau} = -S_D h_1 \varepsilon e^{i\tau} - C_D \frac{d}{dt} (h_1 \varepsilon e^{i\tau}) \quad (31)$$

In the dimensionless form:

$$\bar{F}_D = -\bar{S}_D - i\vartheta \bar{C}_D \quad (32)$$

From the equations (29) and (31), the dimensionless Damping Coefficient  $\bar{C}_D$  and Stiffness Coefficient  $\bar{S}_D$  can be found which is

$$\bar{S}_D = -\text{Re}(\bar{F}_D) \approx -\int_0^1 p_{11} d\bar{x} \quad (33)$$

$$\bar{C}_D = -\text{Im}(\bar{F}_D) \approx -\int_0^1 p_{12} d\bar{x} \quad (34)$$

### 7. CENTRE OF PRESSURE

The centre of pressure of bearing in dimensionless form can be given as:

$$\bar{\bar{x}} = \frac{\int_0^1 \bar{x} p d\bar{x}}{\int_0^1 p d\bar{x}} \quad (35)$$

### 8. RESULTS AND DISCUSSIONS

To study the Non-Newtonian effects on the steady and dynamic characteristics of pivoted curved slider bearing, the numerical results for steady state pressure, load carrying capacity, centre of pressure and coefficients of dynamic Stiffness & Damping have been obtained for the different values of parameter of Pseudoplasticity  $\alpha$  and parameter of slider curvature  $\Delta$  within the valid range of convergence [1,13].

The nature of lubricant is Newtonian for the parameter of Pseudoplasticity  $\alpha = 0$ , Dilatant for  $\alpha < 0$  and Pseudoplastic for  $\alpha > 0$ . The bearing become plane pivoted slider for the curvature parameter  $\Delta = 0$ . For the numerical calculation and the analysis of the various results, the values for the film thickness ratio  $1.2 < r_b < 3.7$ , the slider curvature parameter  $0 < \Delta < 0.83$  [13] and the parameter of Pseudoplasticity  $-0.1 < \alpha <$

0.1 [1,6] have been taken in the present analysis.

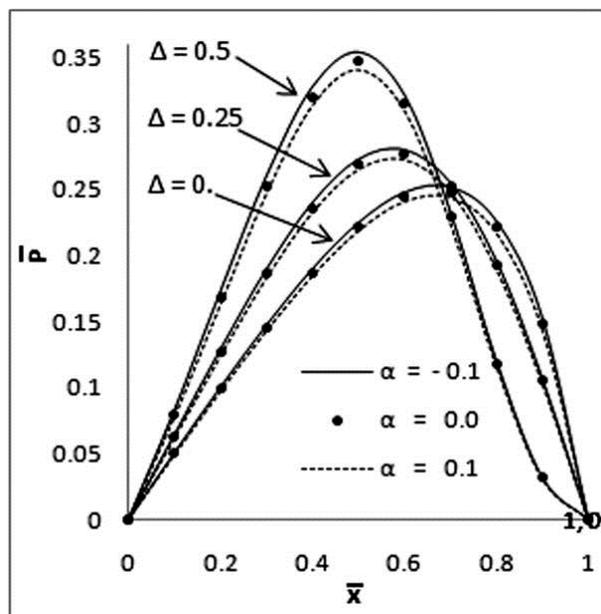


Fig. 2. Variation of dimensionless steady state film pressure ( $\bar{p}_o$ ) with dimensionless coordinate  $\bar{x}$  for  $r_b = 2$ .

Figure 2 shows the variation of dimensionless steady state film pressure with respect to the dimensionless coordinate  $\bar{x}$  for the curvature parameter  $\Delta = 0, 0.25, 0.5$  and  $\alpha = -0.1, 0.0, 0.1$ . It is observed that for each value of  $\Delta$  and  $\bar{x}$ , the dimensionless pressure decreases as  $\alpha$  increases from  $-0.1$  to  $0.1$  i.e. on comparison with the Newtonian case, the dimensionless pressure decreases with the Pseudoplasticity and increases with the Dilatant nature of the lubricant for both the plane and curved slider bearings which agrees with the results of Wada and Hayashi [1] and Hung [6]. Further, the dimensionless pressure is lowest for the plane slider ( $\Delta = 0$ ) and for each value of  $\bar{x}$ , the pressure increases as the curvature increases upto  $\bar{x} \approx 0.7$  and decreases thereafter. Due to this, a shift in the peak value of pressure is observed. This establishes the validity of present analysis for Newtonian lubricants [12,13].

Figure 3 shows the variation of dimensionless steady state load carrying capacity of bearing with respect to the curvature parameter  $\Delta$  with a particular value of step ratio  $r_b = 2$  and different values of Pseudoplasticity parameter  $\alpha$ .

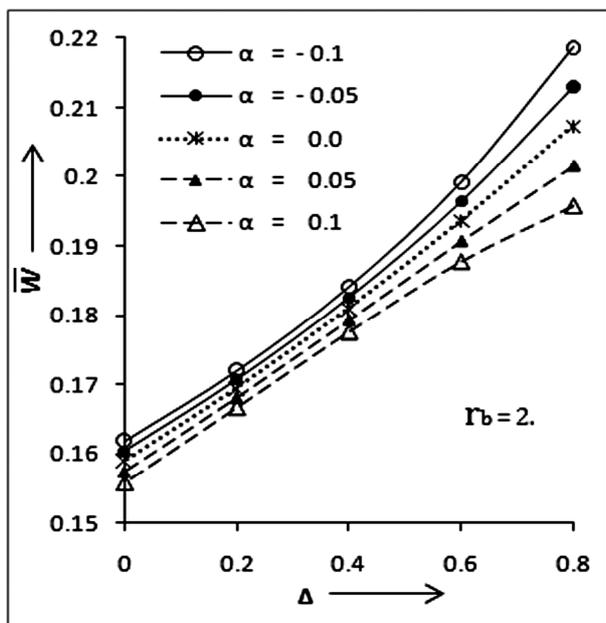


Fig. 3. Variation of dimensionless steady state load carrying capacity ( $\bar{W}$ ) of bearing with  $\Delta$  for  $r_b = 2$ .

It is observed that the dimensionless load capacity increases with the increase of curvature  $\Delta$  which agrees with the results of Kapur<sup>13)</sup> and establishes the present results for Newtonian lubricants ( $\alpha = 0$ ). It is further observed that for each value of  $\Delta$ , the load carrying capacity with Dilatant lubricants ( $\alpha < 0$ ) is higher than that in the Newtonian case and it is less than Newtonian case for Pseudoplastic lubricants ( $\alpha > 0$ ) which is in agreement with real nature of the problem<sup>1,6)</sup>.

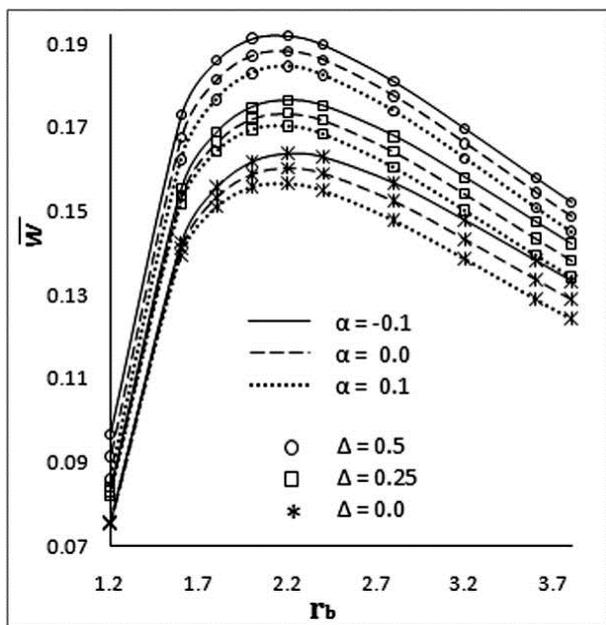


Fig. 4. Variation of dimensionless steady state load carrying capacity ( $\bar{W}$ ) of bearing with  $r_b$ .

Figure 4 shows the variation of dimensionless steady state load carrying capacity of bearing with respect to the step ratio  $r_b$  with different values of curvature parameter  $\Delta$  and Pseudoplasticity parameter  $\alpha$ . It is observed that the dimensionless load capacity increases with the increase in the step ratio  $r_b$  upto  $r_b \approx 2$  and decrease thereafter. It is further observed that for each value of  $\Delta$  and  $r_b$ , the load carrying capacity for  $\alpha = -0.1$  (Dilatant lubricants) is higher than that in the Newtonian case ( $\alpha = 0$ ) and for  $\alpha = 0.1$  (Pseudoplastic lubricants), it is less than Newtonian case. Also, on comparison with the Newtonian case, the deviation of load capacity due to Pseudoplasticity and Dilatant effect is significant with  $r_b > 1.5$ .

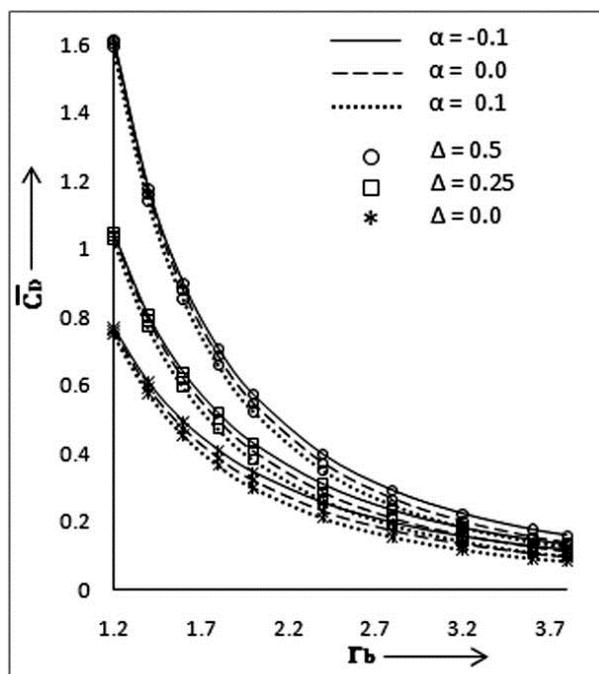


Fig. 5. Variation of dimensionless damping coefficient  $\bar{C}_D$  with thickness ratio  $r_b$ .

Figure 5 shows the variation of dimensionless dynamic Damping Coefficient  $\bar{C}_D$  with respect to the film thickness ratio  $r_b$  for different values of curvature parameter  $\Delta$  and Pseudoplasticity parameter  $\alpha$ . The coefficient of Damping is observed to decrease with the increase in the thickness ratio  $r_b$ . Also, for each value of  $r_b$ , the coefficient of Damping for  $\alpha = -0.1$  is higher than that for  $\alpha = 0$  and for  $\alpha = 0.1$ , it is less than that in the case of  $\alpha = 0$ . Therefore, on comparison with the Newtonian case, the effect of Dilatant fluid increases the

value of Damping coefficient and hence enhances the load capacity, whereas, Pseudoplasticity decreases the value of Damping coefficient. Also, the effect of non-Newtonian (Pseudoplastic and Dilatant) lubricant on damping coefficient is significant with  $r_b > 1.5$ : showing an agreement with the result of load capacity discussed in Fig. 4.

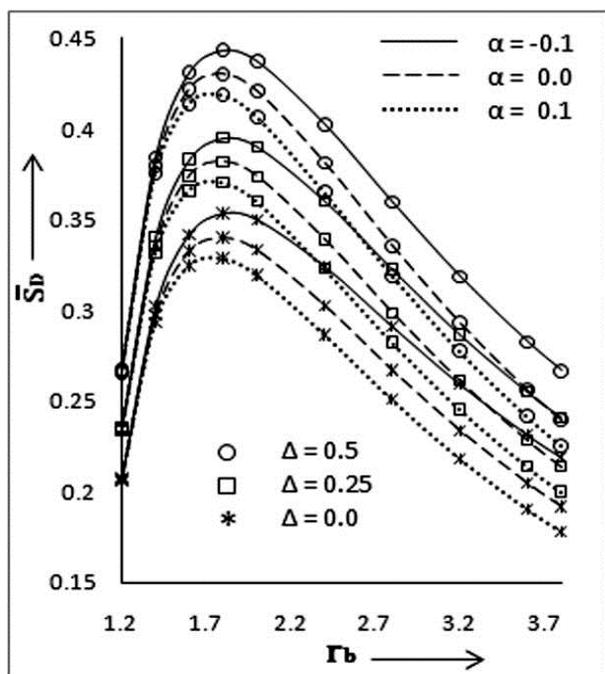


Fig. 6. Variation of dimensionless stiffness coefficient  $\bar{S}_D$  with thickness ratio  $r_b$ .

Figure 6 shows the variation of dynamic Stiffness Coefficient  $\bar{S}_D$  of bearing with respect to the step ratio  $r_b$  with different values of curvature parameter  $\Delta$  and Pseudoplasticity parameter  $\alpha$ . The dynamic Stiffness Coefficient  $\bar{S}_D$  is observed to increase with the increase in the step ratio  $r_b$  up to  $r_b \approx 2$  and decrease thereafter for each  $\Delta$ . It is clearly observed that for each value of  $\Delta$  and  $r_b$ , the Stiffness Coefficient  $\bar{S}_D$  for  $\alpha = -0.1$  (Dilatant lubricants) is higher than Newtonian case ( $\alpha = 0$ ) and for  $\alpha = 0.1$  (Pseudoplastic lubricants), it is less than Newtonian case. Further, the difference in Stiffness coefficient due to non-Newtonian (Pseudoplastic and Dilatant) effects is clearly observed for  $r_b > 1.5$ . However, for  $r_b > 1.5$ , the value of the Stiffness coefficient for Pseudoplastic as well as Dilatant lubricants is of almost same order as for the Newtonian lubricants. Thus, the Dilatant lubricants

significantly increase the life of bearing for  $r_b > 1.5$  and for the Pseudoplastic lubricants, the case is reversed.

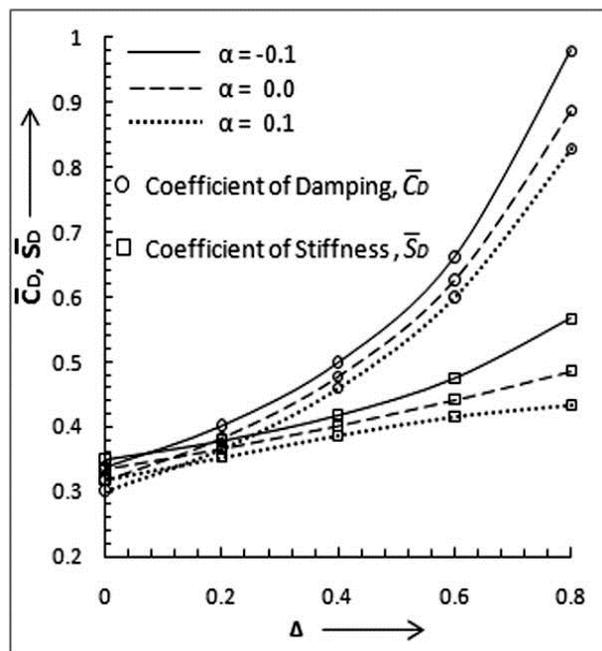
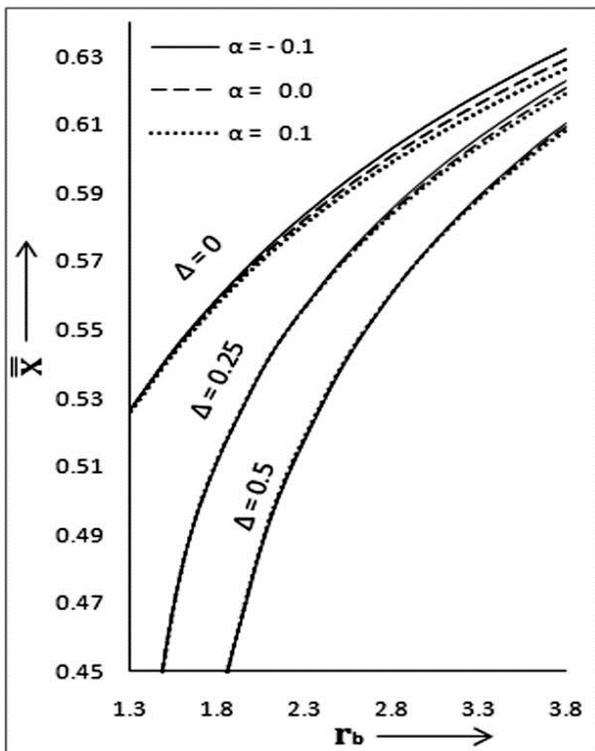


Fig. 7. Variation of dimensionless coefficients of stiffness and damping ( $\bar{S}_D, \bar{C}_D$ ) with  $\Delta$  thickness for ratio  $r_b = 2$ .

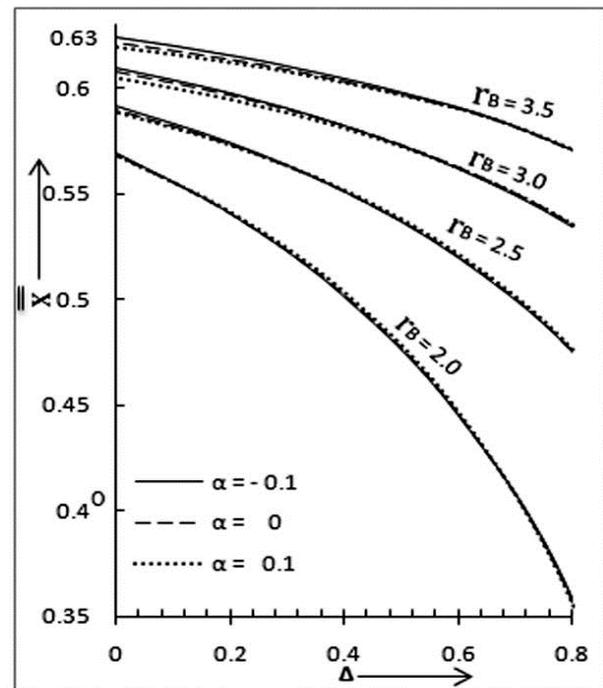
Figure 7 shows the variation of dimensionless dynamic Damping Coefficient  $\bar{C}_D$  and dynamic Stiffness Coefficient  $\bar{S}_D$  with respect to the slider curvature parameter  $\Delta$  for different values of Pseudoplasticity parameter  $\alpha$  with step ratio  $r_b = 2$ . Both the coefficients of Damping and Stiffness show an increase with the increase in  $\Delta$ . Further, for each value of  $\Delta$ , both the coefficients of Damping and Stiffness for  $\alpha = -0.1$  (Dilatant lubricants) is higher than the Newtonian case ( $\alpha = 0$ ) and for  $\alpha = 0.1$  (Pseudoplastic lubricant), it is smaller than the Newtonian case.

Thus the effect of increasing the curvature as well as Dilatant lubricant is observed to increase the Pressure and load capacity. Further, increase in dynamic Damping enhances the bearing stability and hence its performance and increase in the Dynamic Stiffness of the bearing increases the bearing life due to its property. The results of Pseudoplasticity indicate towards the instability and shorter bearing life.



**Fig. 8.** Variation of dimensionless centre of pressure of bearing ( $\bar{x}$ ) with  $r_b$ .

Figure 8 shows the variation of dimensionless centre of pressure  $\bar{x}$  with respect to  $r_b$  ( $1.3 < r_b < 3.8$ ) for curvature parameter  $\Delta = 0, 0.25, 0.5$  with different values of parameter of Pseudoplasticity  $\alpha$ . It is observed that the centre of pressure moves towards the outlet of the bearing with increase of  $r_b$ . Also, it is clear from the figure that the relative movement of Centre of Pressure is enhanced with the increase of  $\Delta$ . Further, for each  $\Delta$ , the plot of Centre of Pressure with  $\alpha = -0.1$  (Dilatant lubricant) is above the Newtonian plot ( $\alpha = 0$ ) and for  $\alpha = 0.1$  (Pseudoplastic lubricant), it is below Newtonian plot i.e. on comparison with the Newtonian case, a shift of the Centre of Pressure towards the inlet of the bearing is observed with the Pseudoplastic lubricants and a shift of the Centre of Pressure towards the outlet of the bearing is observed with the Dilatant lubricants. However, the effect of Non-Newtonian (Pseudoplastic & Dilatant) lubricant on centre of pressure is observed significant in the case of plane slider and it decreases with the increase of curvature parameter  $\Delta$ . Thus, the effect of curvature is analyzed to stabilize the centre of pressure over the lubricant effects.



**Fig. 9.** Percentage variation of dimensionless centre of pressure of bearing with  $\Delta$ .

Figure 9 shows the variation of dimensionless centre of pressure  $\bar{x}$  with respect to the curvature parameter  $\Delta$  for different values of  $\alpha$  and  $r_b$ . It is observed that on increasing the slider curvature  $\Delta$ , the centre of pressure shifts towards the inlet of the bearing for each value of  $r_b$  and each value of  $\alpha$ . The change of centre of pressure due Non-Newtonian (Pseudoplastic & Dilatant) effect is observed to be significant for  $\Delta \leq 0.2$  and  $r_b \geq 3$  and for this range of parameters, the centre of pressure moves towards the inlet with the Pseudoplastic fluids and towards the outlet with the Dilatant fluids. It is clearly observed from the figure that for  $r_b < 3$ , the centre of pressure is not much affected due to the Non-Newtonian (Pseudoplastic & Dilatant) lubricants regardless of the curvature. Again, for  $\Delta > 0.3$ , the stability of centre of pressure is not affected due to lubricants regardless of the value of step ratio  $r_b$ .

Therefore, it is concluded that with a suitable choice of design parameters  $\Delta > 0.2$  and  $2 < r_b < 3$ , the effect of non-Newtonian (Pseudoplastic and Dilatant) lubricants on the shift of centre of pressure can be avoided, and stability of the centre of pressure and hence the stability of the bearing can be improved.

## 9. CONCLUSIONS

The effects of isothermal incompressible non-Newtonian Pseudoplastic and Dilatant lubricants on the steady and dynamic characteristics of one-dimensional pivoted curved slider bearings, neglecting the effects of fluid inertia and cavitation, are presented.

For the Rabinowitsch fluid model, the modified Reynolds equation considering transient motion of the slider is derived. Further, the modified Reynolds equations for the steady state and damping conditions have been obtained.

To obtain the steady and dynamic characteristics of the bearing, the two modified Reynolds equations have been solved using small perturbation technique. The results are in well agreement with the Newtonian results for the coefficient of pseudoplasticity  $\alpha = 0$ .

The steady pressure and steady load, dynamic damping and dynamic Stiffness as well as the centre of pressure and hence the bearing stability, performance and life depend upon the Coefficient of Pseudoplasticity  $\alpha$ , step ratio  $r_b$  and curvature  $\Delta$ .

Based on the results, so obtained, the following conclusions have been drawn:

1. Steady state pressure and load capacity increases significantly with the Dilatant lubricants and curvature, and decreases with the Pseudoplastic lubricants.
2. Dynamic damping and dynamic Stiffness of bearing significantly increase with the Dilatant lubricant as well as the curvature and hence enhances the stability and life of the bearing but the case is reversed for the Pseudoplastic lubricants.
3. The steady state load capacity and dynamic Stiffness increases with  $r_b$  upto  $r_b \approx 2$  and decreases thereafter while the dynamic Damping decreases with the increase of  $r_b$ .
4. An indication of small and less significant non-Newtonian effects on steady and dynamic characteristics is observed for  $r_b < 1.7$ .
5. The centre of pressure moves towards the bearing inlet with the increase of curvature  $\Delta$ , and it moves towards the outlet with

increase of step ratio  $r_b$ .

6. The Pseudoplastic lubricants shift the centre of pressure towards bearing inlet while the Dilatants shift it towards outlet. However, with the suitable choice of design parameters  $\Delta > 0.2$  and  $2 < r_b < 3$ , shift of centre of pressure becomes almost negligible of the non-Newtonian effects.

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- $\bar{x}$  :  $x/B$ ,
- $\alpha$  :  $\kappa\mu^2U^2/h_1^2$ ,
- $\Delta$  :  $H_c/h_1$ ,
- $\varepsilon$  : Small amplitude of oscillation,
- $\kappa$  : Coefficient of Pseudoplasticity,
- $\mu$  : Zero shear rate viscosity,
- $\omega$  : Frequency of oscillation,
- $\tau$  :  $t\omega$ ,
- $\tau_{xy}$  : Shearing stress,
- $\vartheta$  :  $B\omega/U$ .

## NOMENCLATURE

- $B$  : Length of bearing,
- $C_D$  : Dynamic damping coefficient,
- $\bar{C}_D$  :  $h_1^3C_D/\mu LB^3$ ,
- $h, \bar{h}$  : Film thickness,  $\bar{h} = h/h_1$
- $h_m(t)$  : Time dependent film thickness,
- $\bar{h}_m$  :  $h_m/h_1$ ,
- $h_s, \bar{h}_s$  : Steady state film thickness,  $\bar{h}_s = h_s/h_1$
- $h_1, h_2$  : Outlet and Inlet film thickness,
- $F_D, \bar{F}_D$  : Film force,  $F_D = h_1^2 F_D / \mu ULB^2$
- $H_c$  : Height of crown segment of slider,
- $L$  : Width of bearing,
- $p, \bar{p}$  : Film pressure,  $\bar{p} = h_1^2 p / \mu UB$
- $r_b$  : Step ratio  $h_2/h_1$ ,
- $S_D$  : Dynamic stiffness coefficient,
- $\bar{S}_D$  :  $h_1^3 S_D / \mu ULB^2$
- $t$  : Time,
- $u, v$  : Velocity in x and y directions,
- $W, \bar{W}$  : Load capacity,  $\bar{W} = W h_1^2 / \mu ULB^2$