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RESEARCH

Effects of Velocity-Slip and Viscosity Variation in Squeeze Film Lubrication of Two Circular Plates

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A B S T R A C T

A generalized form of Reynolds equation for two symmetrical surfaces is taken by considering velocity-slip at the bearing surfaces. This equation is applied to study the effects of velocity-slip and viscosity variation for the lubrication of squeeze films between two circular plates. Expressions for the load capacity and squeezing time obtained are also studied theoretically for various parameters. The load capacity and squeezing time decreases due to slip. They increase due to the presence of high viscous layer near the surface and decrease due to low viscous layer.

1. INTRODUCTION

In general, most of the lubricated systems can be considered to consist of moving /stationary surfaces (plane/curve, loaded/unloaded) with a thin film of an external material (lubricant) between them. The presence of such a thin film between these surfaces not only helps to support considerable load but also minimizes friction. The characteristics such as pressure in the film, frictional force at the surface, flow rate of the lubricant etc. of the system depend upon the nature of the surfaces, the nature of the lubricant film boundary conditions etc.

The equation governing the pressure generated in the lubricant film can be obtained by coupling the equations of motion with the equation of continuity and was first derived by Reynolds [1] in 1886 and is known as ``Reynolds Equation''. In deriving this equation, the thermal, compressibility, viscosity variation, slip at the surfaces, inertia and surface roughness effects were ignored. Later this Reynolds equation is modified in 1949 by Cope [2] including viscosity and density variation along the fluid film. In 1957-58 the viscosity variation across the film thickness has been considered by Zienkiewicz and Cameron [3,4] who also pointed out that temperature gradient and viscosity variation across the film may not be ignored. In the year 1962, Dowson [5] unified the various attempts in generalizing the Reynolds Equation by considering the variation of fluid properties across as well as along the fluid film thickness by neglecting the slip effects at the bearing surfaces. Since then many workers including myself have studied the effects of viscosity variation in lubricated systems by considering Reynolds Equation with energy equation [6-13]. R.M.Patel et.al [14] studied the performance of a magnetic fluid based squeeze film between transversely rough triangular plates. Also M.E.Shimpi , G.M.Dehari [15] studied surface roughness and elastic deformation effects on the behaviour of the magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets and improved in 2012 to the rotating curved porous circular plates [16].In this study the effects of velocity-slip and viscosity variation in squeeze film lubrication of two circular plates has been discussed.

2. BASIC EQUATIONS

Consider the laminar flow of a fluid between two symmetric surfaces, whose physical configuration is as shown in the Fig. 1. Considering the variation of fluid properties across as well as along the film thickness, the basic equations of motion and equation of continuity in their general form for a newtonian fluid can be written as:

$$\rho\frac{Dv}{Dt} = \rho Y - \frac{\partial P}{\partial y} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z}\right)\right\}$$

$$+\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\} + \frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right\}$$

$$\rho\frac{Dw}{Dt} = \rho Z - \frac{\partial P}{\partial z} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x}\right)\right\} + \frac{2}{3}\frac{\partial}{\partial z}\left\{\eta\left(\frac{\partial w}{\partial z} - \frac{\partial v}{\partial y}\right)\right\}$$

$$+\frac{\partial}{\partial x}\left\{\eta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right\} + \frac{\partial}{\partial y}\left\{\eta\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)\right\} \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (2)$$

with the following usual assumptions of lubrication theory:

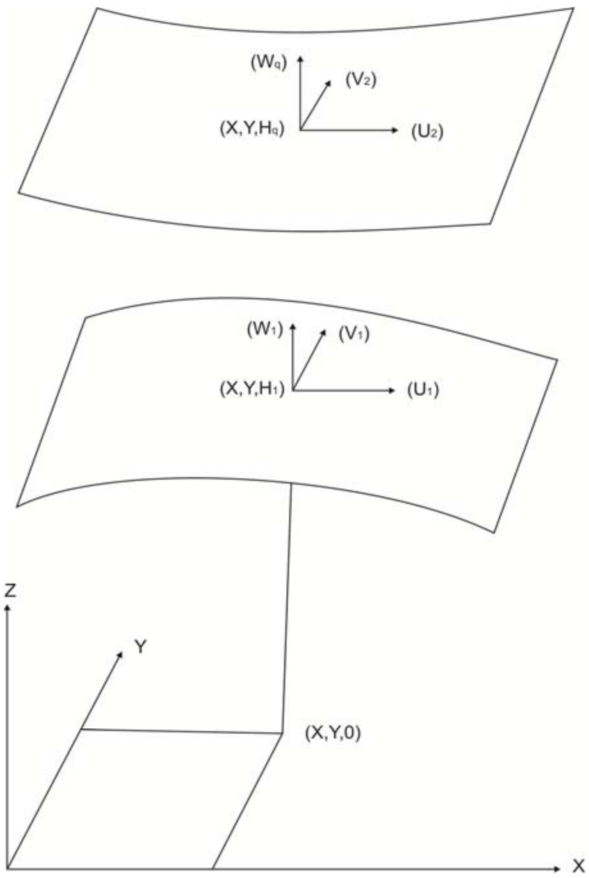

Fig. 1. Coordinate System.

1) Inertia and body force terms are negligible compared with the pressure and viscous terms.
2) There is no variation of pressure across the fluid film, which means $\frac{\partial \rho}{\partial z}$ =0.
3) There is no slip in the fluid-solid boundaries.
4) No external forces act on the film.
5) The flow is viscous and laminar.
6) Due to the geometry of fluid film the derivatives of u and v with respect to z are much larger than other derivatives of velocity components.
7) The height of the film h is very small compared to the bearing length *l*.
   A typical value of h/*l* is about 10^{-3}.

The Navier–Stokes equation (1) can be simplified by Dowson [5] as follows

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial z}\left[\eta \frac{\partial u}{\partial z}\right]$$
$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial z}\left[\eta \frac{\partial v}{\partial z}\right] \quad (3)$$

where P = P (x,y) is the pressure in the film and η is the viscosity.

The boundary conditions considering slip at the surfaces [17] are:

$$u = (u)_1 = (\lambda)_1 \left[\frac{\partial u}{\partial z}\right]_1 + U_1$$
$$\text{at } Z = H_1$$
$$v = (v)_1 = (\delta)_1 \left[\frac{\partial v}{\partial z}\right]_1 + V_1$$

$$u = (u)_2 = -(\lambda)_2 \left[\frac{\partial u}{\partial z}\right]_2 + U_2$$
$$\text{at } Z = H_2$$
$$v = (v)_2 = -(\delta)_2 \left[\frac{\partial v}{\partial z}\right]_2 + V_2 \quad (4)$$

where $(\ )_1$ $(\ )_2$ denote the value at z = H_1 and z = H_2. Here λ's and δ's are molecular mean free path for gas lubrication and depend upon the lubricant temperature, pressure and viscosity. In liquid lubrication λ and δ depend on viscosity and the coefficient is sliding friction. However, with porous bearings λ and δ are functions of slip coefficient at the wall and the permeability parameter of the porous facing.
Integrating equation (3) and using boundary conditions (4) expressions for the fluid film velocities are obtained.

$$u = U_1 + \left[\alpha_1 H_1 + \int_{H_1}^{z} \frac{z dz}{\eta}\right]\frac{\partial P}{\partial x} + \left[\frac{U_2 - U_1}{F_0} - \frac{F_1}{F_0}\frac{\partial P}{\partial x}\right]\left[\alpha_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]$$

$$v = V_1 + \left[\beta_1 H_1 + \int_{H_1}^{z} \frac{z dz}{\eta}\right]\frac{\partial P}{\partial y} + \left[\frac{V_2 - V_1}{F_0^1} - \frac{F_1^1}{F_0^1}\frac{\partial P}{\partial y}\right]\left[\beta_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right] \quad (5)$$

where:

$$F_0 = \alpha_1 + \alpha_2 + \int_{H_1}^{z}\frac{dz}{\eta}, \quad F_0^1 = \beta_1 + \beta_2 + \int_{H_1}^{z}\frac{z dz}{\eta},$$

$$F_1 = \alpha_1 H_1 + \alpha_2 H_2 + \int_{H_1}^{H_2}\frac{z dz}{\eta}, \quad F_1^1 = \beta_1 H_1 + \beta_2 H_2 + \int_{H_1}^{z}\frac{z dz}{\eta},$$

$$\alpha_1 = \frac{(\lambda)_1}{(\eta)_1}, \quad \alpha_2 = \frac{(\lambda)_2}{(\eta)_2}, \quad \beta_1 = \frac{(\delta)_1}{(\eta)_1}, \quad \beta_2 = \frac{(\delta)_2}{(\lambda)_2} \quad (6)$$

Integrating the equation of continuity (2) w.r.t. z. and taking limits from z = H_1 to z = H_2 gives

$$\int_{H_1}^{H_2}\frac{\partial \rho}{\partial t}dz + \int_{H_1}^{H_2}\frac{\partial}{\partial x}(\rho u)dz + \int_{H_1}^{H_2}\frac{\partial}{\partial y}(\rho v)dz + (\rho w)_{H_1}^{H_2} = 0 \quad (7)$$

The integrals of (ρu) and (ρv) are evaluated by partial integration. Introducing the expressions for (ρu) and (ρv) and their derivatives in equation (7) gives:

$$\frac{\partial}{\partial x}\left\{(F_2 + G_1)\frac{\partial P}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{(F_2^1 + G_1^1)\frac{\partial P}{\partial y}\right\} =$$
$$H_2\left\{\frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial y}(\rho v)_2\right\} - H_1\left\{\frac{\partial}{\partial x}(\rho u)_1 + \frac{\partial}{\partial y}(\rho v)_1\right\}$$
$$+ \int_{H_1}^{H_2}\frac{\partial \rho}{\partial y}dz + (\rho w)_{H_1}^{H_2} \quad (8)$$

where

$$F_2 = \int_{H_1}^{H_2}\frac{\rho z}{\eta}\left[z - \frac{F_1}{F_0}\right]dz$$

$$F_2^1 = \int_{H_1}^{H_2}\frac{\rho z}{\eta}\left[z - \frac{F_1^1}{F_0^1}\right]dz, \quad F_3 = \int_{H_1}^{H_2}\frac{\rho z}{\eta}dz$$

$$G_1 = \int_{H_1}^{H_2}\left[z\frac{\partial \rho}{\partial z}\left\{\alpha_1 H_1 + \int_{H_1}^{z}\frac{z dz}{\eta} - \frac{F_1}{F_0}\left[\alpha_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right]dz$$

$$G_1^1 = \int_{H_1}^{H_2}\left[z\frac{\partial \rho}{\partial z}\left\{\beta_1 H_1 + \int_{H_1}^{z}\frac{z dz}{\eta} - \frac{F_1^1}{F_0^1}\left[\beta_1 + \int_{H_1}^{z}\frac{dz}{\eta}\right]\right\}\right]dz$$

$$G_2 = \int_{H_1}^{H_2} \left\{ z \frac{\partial \rho}{\partial z} \left[\alpha_1 + \int_{H_1}^{z} \frac{dz}{\eta} \right] \right\} dz$$

$$G^2{}_1 = \int_{H_1}^{H_2} \left\{ z \frac{\partial \rho}{\partial z} \left[\beta_1 + \int_{H_1}^{z} \frac{dz}{\eta} \right] \right\} dz, G_3 = \int_{H_1}^{H_3} z \frac{\partial \rho}{\partial z} dz \quad (9)$$

Equation (8) represents a generalized form of Reynolds equation for compressible fluid film lubrication considering slip velocities at the bearing surfaces. The two sets of functions F and G depend upon the variation of fluid properties both along as well as across the film and on the slip conditions at the surfaces.

i.e., $(\lambda)_1 = (\lambda)_2 = (\delta)_1 = (\delta)_2 = 0$

$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$

The velocity of the lubricant can vary across the film and may be different near the bearing surfaces owing to the reaction of additives and surfactants with the surfaces [18-20].

Considering a reasonable case where the density and viscosity of the lubricant near the bearing surfaces may be different from the central region, we can have

$$\rho = \rho_1 (x, y), \eta = \eta_1 (x, y) \qquad H_1 \le z \le H_1 + h_1$$
$$\rho = \rho_2 (x, y), \eta = \eta_2 (x, y) \qquad H_1 + h_1 \le z \le H_1 + h_1 + h_2$$
$$\rho = \rho_3 (x, y), \eta = \eta_3 (x, y) \qquad H_1 + h_1 + h_2 \le z \le H_1 + h_1 + h_2 + h_3 \quad (10)$$

This introduces the concept of multiple-layer lubrication. By taking

$$U_1 = U \qquad U_2 = V_1 = V_2 = 0$$
$$\alpha_1 = \beta_1 \qquad \alpha_2 = \beta_2$$
$$\frac{\partial \rho_i}{\partial z} = 0 \qquad i = 1,2,3, \ldots\ldots\ldots\ldots \quad (11)$$

The generalized equation with slip reduces to the following form.

$$\frac{\partial}{\partial x}\left[F_2 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_2 \frac{\partial P}{\partial y}\right] = H_2 \left\{ \frac{\partial}{\partial x}(\rho u)_2 + \frac{\partial}{\partial y}(\rho v)_2 \right\} - H_1 \left\{ \frac{\partial}{\partial x}(\rho u)_1 + \frac{\partial}{\partial y}(\rho v)_1 \right\} + U \frac{\partial}{\partial x}\left[\frac{F_3}{F_0}\right] + [\rho w]_{H_1}^{H_2} \quad (12)$$

where:

$$F_0 = \alpha_1 + \alpha_2 + \frac{h_1}{\eta_1} + \frac{h_2}{\eta_2} + \frac{h_3}{\eta_3}$$

$$F_1 = \alpha_1 H_1 + \alpha_2 H_2 + \frac{h_1 (2H_1 + h_1)}{2\eta_1} + \frac{h_2 (2H_1 + 2h_1 + h_2)}{2\eta_2} + \frac{h_3 (2H_1 + 2h_1 + 2h_2 + h_3)}{2\eta_2}$$

$$F_2 = \frac{\rho_1}{3\eta_1}\left\{(H_1 + h_1)^3 - H_1^3\right\} + \frac{\rho_2}{3\eta_2}\left\{(H_1 + h_1 + h_2)^3 - (H_1 + h_1)^3\right\} + \frac{\rho_3}{3\eta_3}\left\{H_2^3 - (H_1 + h_1 + h_2)^3\right\} - \frac{F_1 F_3}{F_0}$$

$$F_3 = \frac{\rho_1 h_1}{2\eta_1}(2H_1 + h_1) + \frac{\rho_2 h_2}{2\eta_2}(2H_1 + 2h_1 + h_2) + \frac{\rho_3 h_3}{2\eta_3}(2H_1 + 2h_1 + 2h_2 + h_3)$$

$$(\rho u)_1 = \rho_1 \alpha_1 \left[H_1 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_1 U\left[1 - \frac{\alpha_1}{F_0}\right]$$

$$(\rho u)_2 = -\rho_3 \alpha_2 \left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial x} + \rho_3 U \frac{\alpha_2}{F_0}$$

$$(\rho v)_1 = \rho_1 \alpha_1 \left[H_1 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial y}$$

$$(\rho v)_2 = -\rho_3 \alpha_2 \left[H_2 - \frac{F_1}{F_0}\right]\frac{\partial P}{\partial y} \quad (13)$$

$$[\rho w]_{H_1}^{H_2} = (\rho u)_2 \frac{\partial H_2}{\partial x} + (\rho v)_2 \frac{\partial H_2}{\partial y} - (\rho u)_1 \frac{\partial H_1}{\partial x} - (\rho v)_1 \frac{\partial H_1}{\partial y} - V_s$$

here V_s is the resultant velocity towards the film. To see the effect of slip, consider three symmetrical incompressible layers between two solid boundaries.

$$\eta_1 = \eta_2 \qquad \rho_1 = \rho_2 = \rho_3$$

$H_1 = 0$ $H_2 = (h+a) = h$, $h_1 = h_3 = a/2$, $h_2 = (h-a)$

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1/\beta \tag{14}$$

may be considered. The Reynolds equation can be written from equation (12) as follows:

$$\frac{\partial}{\partial x}\left[F_4 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[F_4 \frac{\partial P}{\partial y}\right] = U\frac{\partial}{\partial x}(h) - V \tag{15}$$

where

$$F_4 = \frac{(h-a)^3}{12\eta_2} + \frac{a^3 + 3a^2(h-a) + 3a(h-a)^2}{12\eta_1} + \frac{h^2}{2\beta}$$

taking $\beta = \frac{\eta_1}{\lambda}$ as the slip parameter.

3. SQUEEZE FILM LUBRICATION OF TWO CIRCULAR PLATES:

Consider the squeeze film lubrication between two parallel circular plates as shown in Fig. 2. Let the film thickness of the lubricant present between the two plates be `h' and squeeze velocity be `V'.

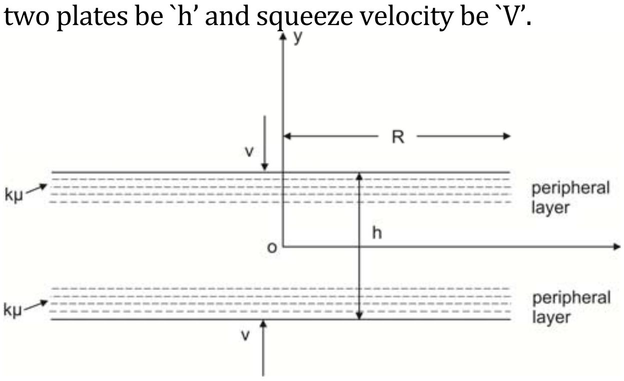

Fig. 2. Squeeze film between two Circular Plates.

The governing equation of flow of the lubricant in the case of squeeze film lubrication is given by equation [15] as:

$$\frac{d}{dx}\left[F_4 \frac{dP}{dx}\right] = -V \tag{16}$$

where $F_4 = \frac{1}{12\mu}\left[\frac{(h-a)^3(k-1)+h^3}{k} + \frac{6h^2}{\beta}\right]$

where h is the total film thickness, a is the thickness of the peripheral layer, k is the ratio of the viscosities, μ be the viscosity of the base lubricant i.e., the middle layer, β be the slip parameter.

The equation (16) can be written in the following form:

$$\frac{d}{dx}\left[F_4 \frac{dP}{dx}\right] = -V \tag{17}$$

where $F_4 = \frac{l^3}{12\mu}\left[\frac{(\bar{h}-\bar{a})^3(k-1)+\bar{h}^3}{k} + \frac{6\bar{h}^2}{\bar{\beta}}\right]$

and $$\bar{a} = \frac{a}{l};\ \bar{h} = \frac{h}{l};\ \bar{\beta} = \frac{\beta}{\left(\frac{\mu}{l}\right)} \tag{18}$$

The flow flux, Q of the lubricant is given by equation (17) as

$$Q = 2b\left[F_4 \frac{dP}{dx}\right] \tag{19}$$

where F_4 is given by the equation(18) and b is the width of the bearing. In the case of circular plates b is equal to $2\pi r$.

The flux Q obtained from the equation of continuity is given by

$$Q = 4\pi r^2 V \tag{20}$$

Now from equations (19) and (20), we obtain

$$\frac{dP}{dr} = \frac{-Vr}{F_4} \tag{21}$$

The boundary condition for equation (21) is

$$P = 0 \qquad \text{at } r = R$$

Now using the above condition and integrating equation (21), we get

$$P = \frac{V}{2F_4}\left[R^2 - r^2\right] \tag{22}$$

where R is the radius of the approaching surfaces.

The load capacity W is given by

$$W = \int_0^R (2\pi r) P dr \tag{23}$$

substituting equation (22) in (23), we get

$$W = \frac{\pi V}{F_4} \frac{R^4}{4} \tag{24}$$

The squeezing time, T is given from (24) as

$$T = \frac{\pi R^4}{4W} \int_{h_f}^{h_i} \frac{dh}{F_4} \tag{25}$$

where h_i is the initial film thickness and h_f is the final film thickness and F_4 is given by

$$F_4 = \frac{1}{12\mu}\left[\frac{(h-a)^3(k-1)+h^3}{k} + \frac{6h^2}{\beta}\right]$$

Now the equations (24) and (25) are non-dimensionalised as given below and numerically analyzed to see the effects of velocity-slip and viscosity variation. Similar results can be expected for the case of parallel plates.

Equations (24) and (25) are non-dimensionalised in the following manner:

$$\bar{a} = \frac{a}{l} \; ; \; \bar{h} = \frac{h}{l} \; ; \; \bar{\beta} = \frac{\beta}{\left(\frac{\mu}{l}\right)} \; ; \; \bar{V} = \frac{V\mu}{P_0 l} \; ;$$

$$\bar{h_f} = \left(\frac{h_f}{l}\right); \; \bar{F_4} = \frac{F_4}{\left(\frac{l^3}{12\mu}\right)}; \; \bar{h_i} = \left(\frac{h_i}{l}\right)$$

Thus

$$\bar{W} = \frac{W}{P_0 l} = 4\frac{\bar{V}}{\bar{F_4}} \tag{26}$$

and
$$\bar{T} = \frac{T}{\left(\frac{4\mu l}{W}\right) h_f} = \int_1^{\bar{h}_{i1}} \frac{d\bar{h}}{\bar{F_4}} \tag{27}$$

where
$$\bar{F_4} = \left[\frac{(\bar{h}-\bar{a})^3(k-1)+\bar{h}^3}{k} + \frac{6\bar{h}^2}{\bar{\beta}}\right] \tag{28}$$

equations (26) and (27) are analyzed numerically and graphs have been plotted.

4. RESULTS AND DISCUSSIONS

a) Load Capacity:

The parameters considered here are $\bar{\beta}, k$ and $\bar{a}$. So $\bar{\beta}$ represents the slip, k represents the ratio of the viscosities of the peripheral layer to the middle layer and $\bar{a}$ be the thickness of the peripheral layer. $\bar{\beta}$ represents the non-dimensionalised slip parameter. Low values of $\bar{\beta}$ indicates high slip at the surfaces and as $\bar{\beta}$ increases the slip decreases and it tends to zero for high values of $\bar{\beta}$. Thus an increases in $\bar{\beta}$ indicates decreasing the slip at the surfaces.

In Figs. 3-5, the load capacity, $\bar{W}$ is plotted w.r.t $\bar{\beta}$ for various values of k treating $\bar{a}$ as constant. All these graphs coincides for $k = 1$.

It is seen from these figures that the load capacity increases as $\bar{\beta}$ increases indicating that the load capacities decrease due to slip and decreases further as the slip parameter increases. It is also seen from these graphs that the load capacities increase due to increase in the value of k that is the peripheral layer viscosity i.e., the capacity increase as the peripheral layer viscosity increases.

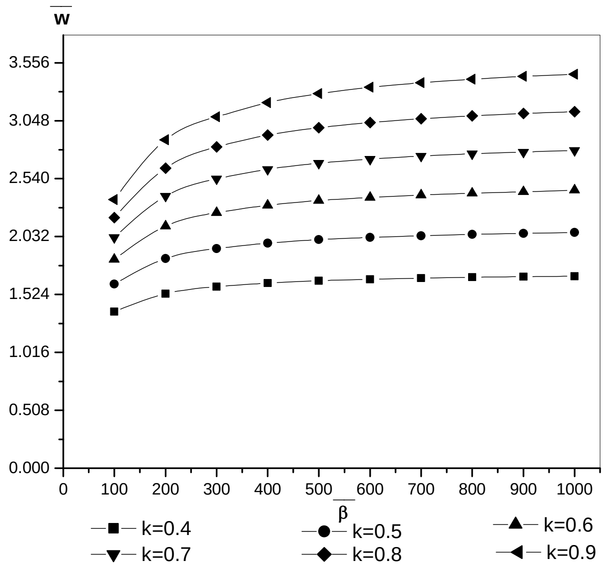

Fig. 3. Variation of $\bar{W}$ with $\bar{\beta}$ for various values of k.

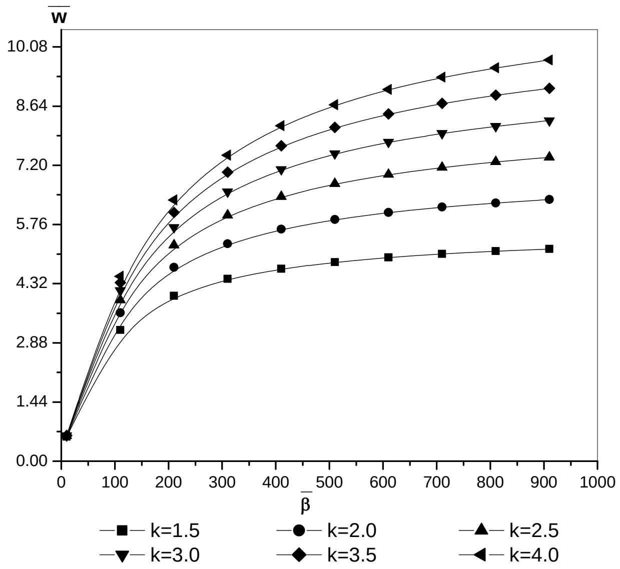

Fig. 4. Variation of $\overline{W}$ with $\overline{\beta}$ for various values of k .

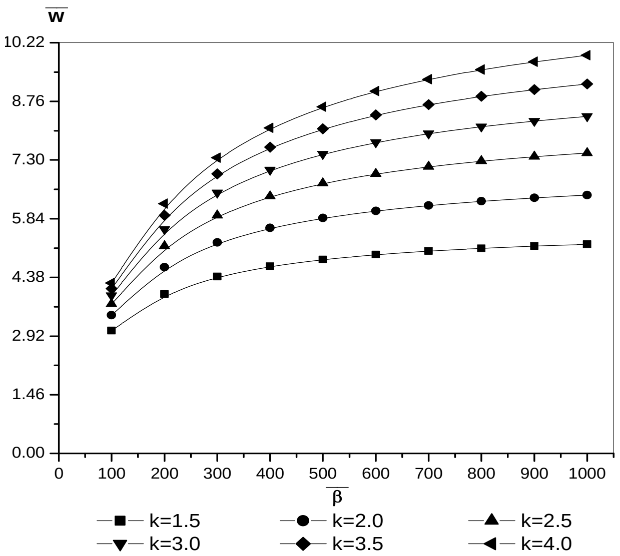

Fig. 5. Variation of $\overline{W}$ with $\overline{\beta}$ for various values of k .

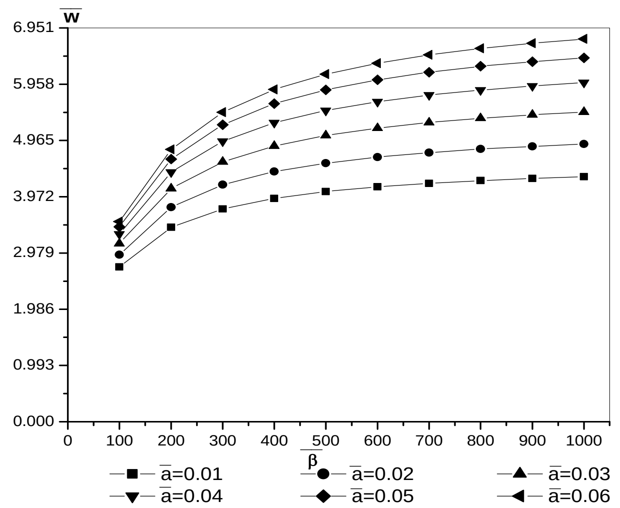

Fig. 6. Variation of $\overline{W}$ with $\overline{\beta}$ for various values of $\overline{a}$.

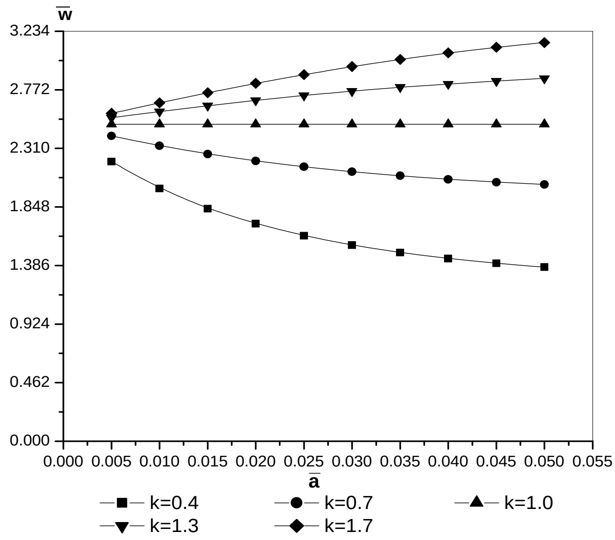

Fig. 7. Variation of $\overline{W}$ with $\overline{a}$ for various values of k .

In Fig. 6, the load capacity is plotted with $\overline{\beta}$ for various values of $\overline{a}$ (for $k > 1$). It is seen from these figures that the load capacity decreases as the slip increases and they increase as the peripheral layer increases.

In Fig. 7, the load capacity is plotted with $\overline{a}$ for various k. It is seen from the graph that for $k = 1$, it is parallel to x-axis. That is when the peripheral layer viscosity is same as the middle layer viscosity, the effect of increase in the peripheral layer is nil as expected. It is also seen from the graph, that when $k < 1$, the load capacity decrease as the peripheral layer viscosity increases i.e., as $\overline{a}$ increases.

That is when the peripheral layer viscosity is less than the middle layer viscosity, the load capacity decreases as the thickness peripheral layer increases. It is also seen from the graph that when $k > 1$, the load capacity increase as the peripheral layer viscosity increases indicating that when the peripheral layer viscosity is higher than the middle layer, the load capacity increases and this increase is enhanced as the thickness of the peripheral layer increases. It is in agreement with the experimental reports observed by various works of Cameron etc. [17] that when high polymer additives are added to the base lubricant, the lubricant properties improved. The high polymer additives due to their affinity towards the surface attach themselves to the surface and form a high viscous layer near the surface, that is the case of $k > 1$, where we observed increase in the load capacity.

In Figs. 8 and 9, the load capacity is plotted with k for various $\overline{a}$. It is found these figures, that the load capacity increases, as k increases for $k > 1$ and it is more for higher values of $\overline{a}$ as expected from the previous results.

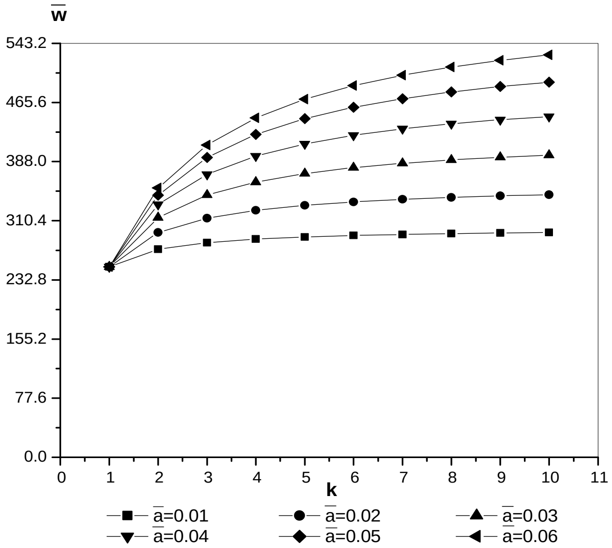

Fig. 8. Variation of $\overline{W}$ with k for various values of $\overline{a}$.

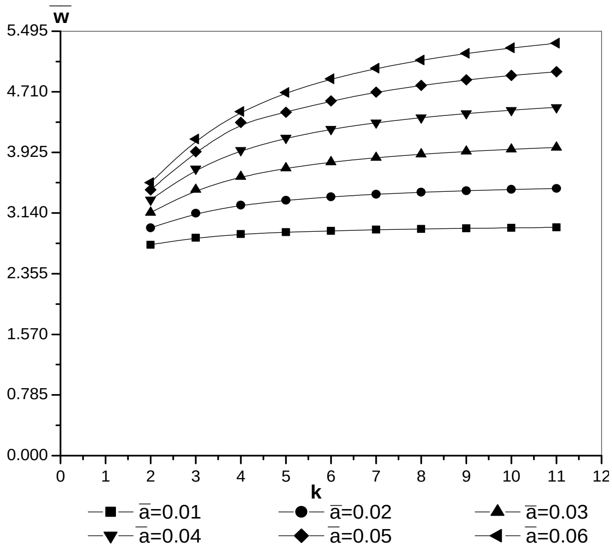

Fig. 9. Variation of $\overline{W}$ with k for various values of $\overline{a}$.

b) Squeezing Time:

Equation (27) is integrated numerically for various values of $\overline{\beta}$, k, $\overline{a}$ and graphs have been plotted for squeezing time with various values of these parameters in Figs. 10-14.

In the Figs. 10 and 11, squeezing time, $\overline{T}$ is plotted with $\overline{\beta}$ for various k. It is found from these figures that the squeezing time increases as $\overline{\beta}$ increases, that as slip parameter increase. It is mentioned earlier that the slip decreases as $\overline{\beta}$ increases. Thus due to slip the squeezing time decreases and decreases further as the slip increases. It is also observed from these figures that the squeezing time is more for higher values of k showing that the squeezing time increases as the viscosity of the peripheral layer increases.

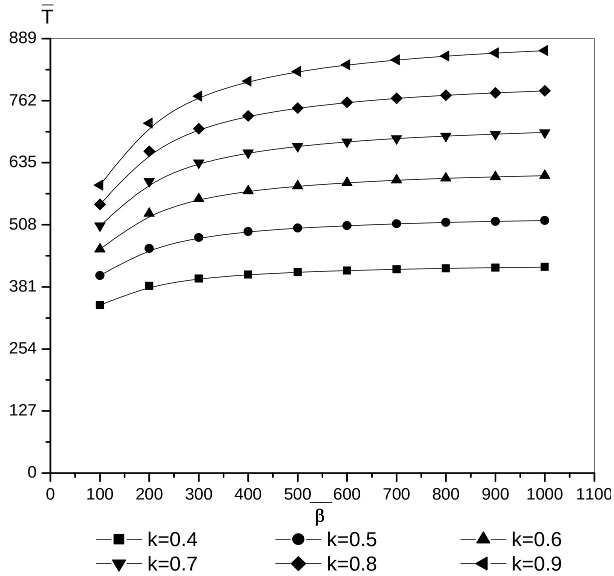

Fig. 10. Variation of $\overline{T}$ with $\overline{\beta}$ for various values of k.

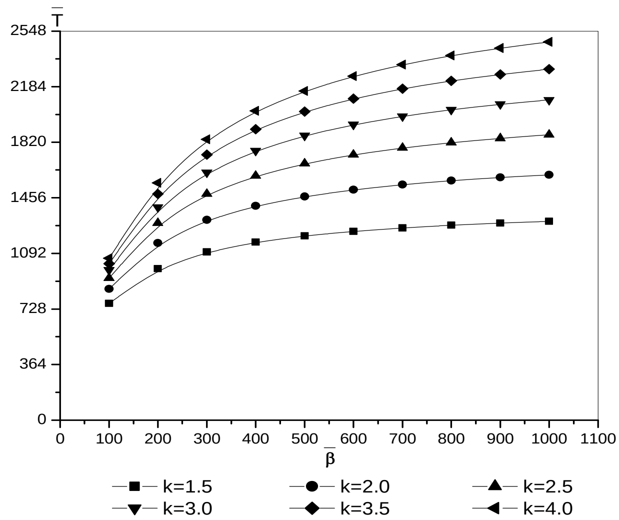

Fig. 11. Variation of $\overline{T}$ with $\overline{\beta}$ for various values of k.

In Fig. 12, the squeezing time, $\overline{T}$ is plotted with $\overline{\beta}$ for various values of $\overline{a}$ taking $k = 2.0$. It is seen from these graphs that the squeezing time increases as $\overline{\beta}$ increases, i.e., as the slip decreases and it has more value for higher values of $\bar{a}$, showing that the squeezing time decreases as the slip increases. It is also observed that for $k > 1$, the squeezing time has more value for higher values of $\bar{a}$, that is when the viscosity of the peripheral layer is more than the middle layer, the squeezing time increases and this increase is enhanced as its thickness increases.

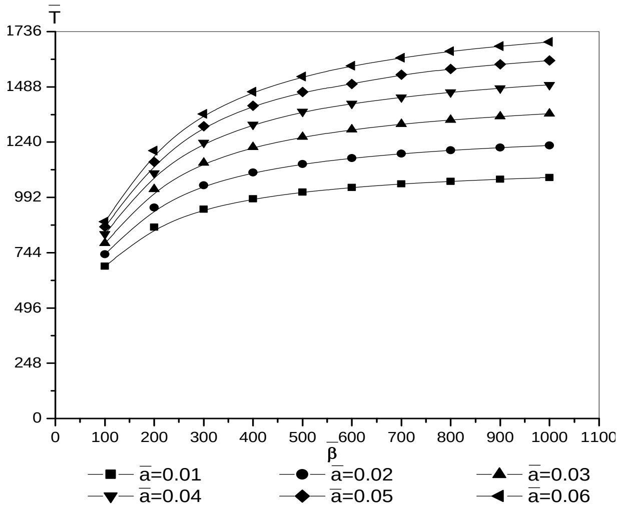

Fig. 12. Variation of $\overline{T}$ with $\overline{\beta}$ for various values of $\bar{a}$.

In Figs. 13 and 14, the squeezing time, $\overline{T}$ is plotted with $\bar{a}$ for various values of k. It is seen from this figure that when $k = 1$, the graph is parallel to the x-axis, that is when the viscosity of the peripheral layer and middle layer are equal, it has no effect on squeezing time as the peripheral layer thickness increases. It is also observed that when $k < 1$, the squeezing time decreases, as $\bar{a}$ increases for $k < 1$ and increases for $k > 1$.

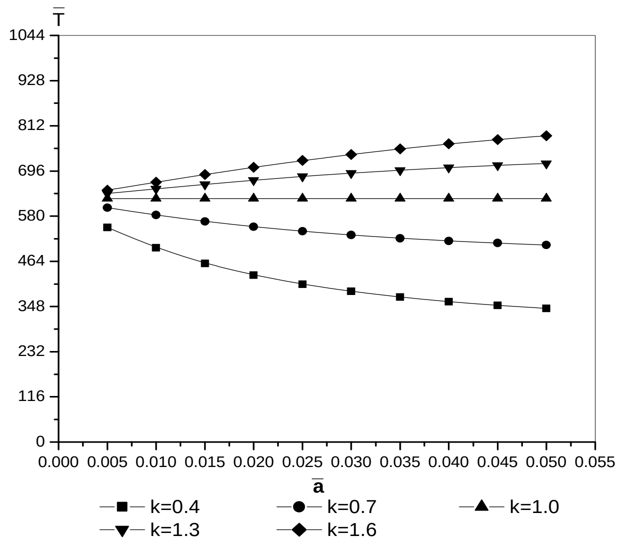

Fig. 13. Variation of $\overline{T}$ with $\bar{a}$ for various values of k.

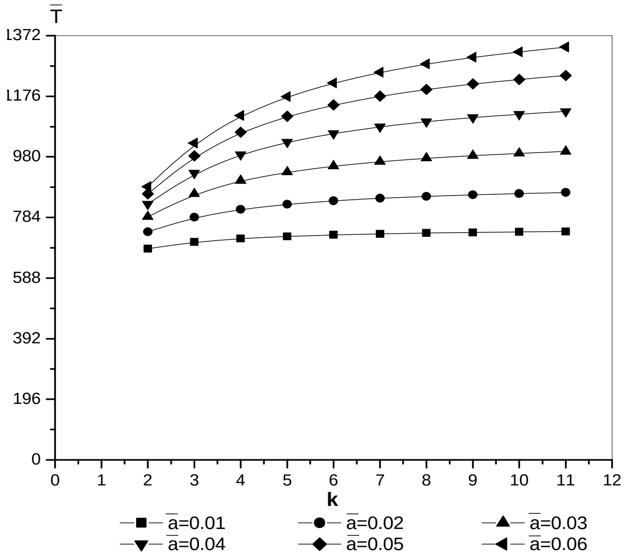

Fig. 14. Variation of $\overline{T}$ with k for various values of $\bar{a}$

That is when the viscosity of the peripheral layer is less than the viscosity of the middle layer, the squeezing time decreases as its thickness increases.

On the other hand, when the viscosity of the peripheral layer is more than the viscosity of the middle layer, the squeezing time increases as its thickness increases. It is in agreement with the experimental reports observed by various workers.

4. CONCLUSION

A generalized form of Reynolds equation applicable to fluid film lubrication was derived considering the variation of fluid properties, both across and along the film thickness, with velocity-slip at the bearing surfaces. The effects of velocity-slip and viscosity variation in squeeze film lubrication of two circular plates have been studied. The beneficial result for hydrodynamic lubrication due to the presence of increased viscosity near the bearing surface was indicated.

However, although the effects of velocity-slip at the bearing is to decrease both the frictional force and the load capacity, the coefficient of friction increases, which leads to an unfavorable results. For a gas-lubricated hydrostatic bearing, the gas film pressure and load decrease with increasing molecular mean free path.

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Nomenclature

- h Total film thickness
- h_f Final film thickness
- k Ratio of the viscosities
- l Length of the bearing
- P Hydrodynamic Pressure
- R Radius of the surfaces in case of circular plates
- T Squeezing time of for stiff surfaces
- V Squeeze Velocity
- W Load capacity for stiff surfaces
- μ Viscosity of the purely hydrodynamic zone

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