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RESEARCH

Damping Characteristics of Polymer-Concrete Outrigger

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A B S T R A C T

The new technology is developed for polymer concrete (PC) structure, which is intended for using as construction materials, for example the bodies of metal-working machines. A mechanism for comparison of materials with different PC structures with good damping characteristics is found. The cross vibration of PC outrigger was investigated as continuum media under the force impulse action in the end of the beam. Analytical solutions about continuous impulse of force are derived and numerical results are presented. The established model allows identification of the elasticity module of the beam material.

1. INTRODUCTION

The creation and wide application of new materials as polymer concrete (PC) in building and mechanical engineering domains requires determination of their various mechanical characteristics. In the last decade there are many papers [1-9] in the field of damping characteristics of this kind of materials, as often the objects of study are log scale coefficient of attenuation δ and dispersing coefficient ψ .

In [9] is developed a new technology for polymer concrete structures, which is intended for using as construction materials, for example the bodies of metal-working machines. For the presented there problem, it is necessary to build up a model for comparison of materials with different PC structures compiled with the aim of reaching a good damping characteristics. The experiments are made on prismatic bars with dimensions in mm *30x30x350* [9]. The beam is stuck vertically and at the initial moment of time at the end of the beam there is a percussion action through a special hammer. Special equipment measures the vibration in the final section after the impact. It is considered the transverse oscillation of the console beam as a continuous medium under the action of hammer strike at the end of the beam and inelastic resistance in the form of surface tension $\vec{p}_\beta$ with a magnitude proportional to speed and backward oriented: $\vec{p}_\beta = -\beta \vec{v}$. The introduced here coefficient of resistance β depends on the method of attachment of the beam, the shape and dimensions, internal friction in the material and the resistance of the air.

2. FORMULATION OF THE PROBLEM

Must be find mechanism for the comparison of damping characteristics of prismatic PC outriggers. The transverse oscillation of the console beam under the action of initial shock impulse and inelastic resistance must be investigated. The following tasks will be considered:

- Determination of the differential equation for transverse oscillations of console beam under the action of impact force $F(t),\ 0 < t \le t_0$, which is applied in the end of beam at point $x = l_{sh}$;
- Determination of the transverse damped natural oscillations of the beam $w(x,t)$;
- Determination of transfer function $H(\omega)$ and response function $h(t)=F^{-1}[H(\omega)]$ for the final point;
- Overcoming the problems associated with instability at determination of response function by experiment.

3. TRANSVERSE OSCILLATIONS OF CONSOLE BEAM AT TRANSVERSE IMPACT LOAD IN THE ENDPOINT

The characteristics of the polymerconcrete beam are following: length l_e, side of the square cross-section a, density ρ_{PB} and Young's modulus E.

The differential equation of transverse oscillations of console beam after transverse impact load at the end is [10]:

$$Dw \equiv \frac{\partial^2 w(\xi,t)}{\partial t^2} + 2n\frac{\partial w(\xi,t)}{\partial t} + k^2\frac{\partial^4 w(\xi,t)}{\partial \xi^4} = f(t)\delta(\xi-\xi_{sh}), \quad (1)$$

$$0 \le \xi \le 1, \quad \xi = x/l_e, \quad t > 0,$$

where:

$$M_\Sigma = \rho_{PB}.a^2.l_e, kg;\ 2n = \frac{\beta a l_e}{M_\Sigma}, \frac{1}{s};\ k^2 = \frac{EI_y}{M_\Sigma l_e^3}, \frac{1}{s^2};\ f(t) = \frac{F(t)}{M_\Sigma}, \frac{m}{s^2}.$$

The concentrated impact force $F(t)$ at point $x = l_{sh}$ is represented as distributed load of the section $|x - l_{sh}| \le \varepsilon$ with intensity:

$$q_z(x,t) = \frac{F(t)}{2\varepsilon} \equiv F(t)\delta_\varepsilon(x-l_{sh}), \frac{N}{m},\ \delta_\varepsilon(x-l_{sh}) = \begin{cases} 1/(2\varepsilon), & x = l_{sh}, \\ 0, & x \ne l_{sh}, \end{cases}$$

$$\int_0^{l_e} \delta_\varepsilon(x-l_{sh})dx = 1,$$

where $\delta(x-l_{sh})$ is the impulse function of first order, as its mechanical mean is infinity force with single impulse.

The boundary and initial conditions (at $t=0$ the beam is in a rest) for Eqn (1) are respectively:

$$w(0,t) = w'(0,t) \equiv 0, \quad (2)$$
$$M_y(l_e,t) = -EI_y w''(l_e,t) \equiv 0,\ Q_z(l_e,t) = \frac{dM_y(l_e,t)}{dx} = -EI_y w'''(l_e,t) \equiv 0,$$

$$w(x,0) \equiv 0, \quad \dot{w}(x,0) = 0, \quad (3)$$

For the solution of homogeneous Eqn (1) is used the Fourier method, i.e. separation of variables:

$$w(\xi,t) = A(t)Z(\xi),\ t > 0,\ 0 \le \xi \le 1, \quad (4)$$

where $A(t)$ is an amplitude depending on the time and the type of load; $Z(\xi)$ is the function of the form of beam deflection for the given fixing and it must be satisfied the boundary conditions. After substitution of (4) in the homogeneous equation (1) and separation of the variables are obtained the following two equations:

$$\ddot{A}(t) + 2n\dot{A}(t) + k^2\lambda^4 A(t) = 0, \quad (5)$$

$$Z^{(4)}(\xi) - \lambda^4 Z(\xi) = 0,\ Z(0) = Z'(0) = Z''(1) = Z'''(1) = 0. \quad (6)$$

The solution of homogeneous equation (6)

$$Z(\lambda\xi) = C_1\sin(\lambda\xi) + C_2\cos(\lambda\xi) + C_3 sh(\lambda\xi) + C_4 ch(\lambda\xi)$$

leads to the search for non-trivial solution satisfying the boundary conditions, i.e. come down to the task of eigenvalues:

$$\cos(\lambda)ch(\lambda) + 1 = 0. \quad (7)$$

The roots of the characteristic equation (7) are:

$$\lambda_1 = 1.875,\ \lambda_2 = 4.694, \lambda_3 = 7.854, ..., \lambda_i = (\pi/2)(2i-1), i > 3.$$

For $\forall\lambda_i$ corresponds to an eigenfunction:

$$Z_i(\lambda_i\xi) = \sin(\lambda_i\xi) - a_i\cos(\lambda_i\xi) + [-sh(\lambda_i\xi) + a_i ch(\lambda_i\xi)],$$
$$a_i = (\sin\lambda_i + sh\lambda_i)/(\cos\lambda_i + ch\lambda_i).$$

For $\forall\lambda_i$ corresponds a solution of Eqn(5) in form:

$$A_i(t) = \bar{A}_i e^{-nt}\sin(t\sqrt{k^2\lambda_i^4 - n^2} + \alpha_i),\ t > 0,,$$

where $\bar{A}_i, \alpha_i$ are constants.

Homogeneous solution of Eqn (1) due to linearity and homogeneity represents a sum of the particular solutions $\{A_i(t)Z_i(\lambda_i\xi)\}_{i=1}^{\infty}$:

$$w_h(\xi,t)=e^{-nt}\sum_{i=1}^{\infty}\overline{A}_i\sin(t\sqrt{k^2\lambda_i^4-n^2}+\alpha_i)Z_i(\lambda_i\xi). \quad (8)$$

Finding a particular solution $w^*(\xi,t)$ of the non-homogeneous differential equation (1) is associated with difficulties and therefore an approximate solution by Bubnov–Galerkin method is used. For this purpose, looking for a solution in the form of series:

$$w^*(\xi,t)\equiv\sum_{i=1}^{n^*}w_i^*(\xi,t)=\sum_{i=1}^{n^*}A_i^*(t)Z_i(\lambda_i\xi),\ t>0,\ 0\le\xi\le1, \quad (9)$$

where $A_i^*(t)$ is an undetermined coefficient representing the amplitude of oscillation, depending on the time and load; $Z(\xi)$ - the function of the form of beam deflection for the given fixing and it must be satisfied the boundary conditions (2).

For the function (9) the requirement is to satisfy the following equation:

$$\int_0^1[Dw^*-f(t)\delta(\xi-\xi_{sh})]\delta w^*(\xi,t)d\xi=0 \quad (10)$$

For the variation δw^* of the function w^* is valid the following: $\delta w^*=\sum_{i=1}^{n^*}Z_i(\lambda_i\xi)\delta A_i^*$, as the functions $Z_i(\lambda_i\xi)$ are fixed.

From Eqn (10) due to the independence of variations follows:

$$\int_0^1[Dw^*-f(t)\delta(\xi-\xi_{sh})]\delta w^*(\xi,t)d\xi=0. \quad (11)$$

The substitution of function w^* from (9) in (11) yields a system with n^* equations for founding of n^* unknowns A_i^*. The i-th term is represented as:

$$\int_0^1\{\sum_{j=1}^{n^*}[\ddot{A}_j^*Z_j+2n\dot{A}_j^*Z_j+k^2A_j^*Z_j^{(4)}]-f(t)\delta(\xi-\xi_{sh})\}Z_i(\xi)d\xi=0.$$

Since the functions Z_i are independent and orthogonal then can be written:

$$\ddot{A}_i^*\|Z_i\|^2+2n\dot{A}_i^*\|Z_i\|^2+k^2\lambda_i^4A_i^*\|Z_i\|^2-f(t)Z_i(\xi_{sh})=0,\ i=1,..,n^*,$$

$$\|Z_i\|^2=\int_0^1Z_i^2(\xi)d\xi,\quad \int_0^1Z_i^{(4)}(\xi)Z_i(\xi)d\xi=\lambda_i^4\|Z_i\|^2,$$

$$\int_0^1Z_j(\xi)Z_i(\xi)d\xi=\delta_{ij}\|Z_i\|^2,\quad \delta_{ij}=\begin{cases}1, i=j,\\0, i\ne j,\end{cases}\quad Z_i(\xi_{sh})=\int_0^1\delta(\xi-\xi_{sh})]Z_i(\xi)d\xi.$$

After simplification the following equation is obtained (at zero initial conditions):

$$\ddot{A}_i^*+2n\dot{A}_i^*+k^2\lambda_i^4A_i^*-f(t)Z_i(\xi_{sh})/\|Z_i\|^2=0 \quad (12)$$

The Eqn (12) is integrated by the operational methods. For this purpose the imaging equation is constructed:

$$\overline{A_i^*(s)}(s^2+2ns+k^2\lambda_i^4)=[Z_i(\xi_{sh})/\|Z_i\|^2]\overline{f(s)} \quad (13)$$

where $\overline{A_i^*(s)}=L[A_i^*(t)]$, $\overline{f(s)}=L[f(t)]$ are Laplace images of the originals.

For the image of the original is found:

$$\overline{A_i^*(s)}=Z_i(\xi_{sh})\|Z_i\|^{-2}(s^2+2ns+k^2\lambda_i^4)^{-1}\overline{f(s)}\equiv\overline{H_i(s)}.\overline{f(s)}. \quad (14)$$

The image of the original $A_i^*(t)$ is multiplication of the transfer function $\overline{H_i(s)}=Z_i(\xi_{sh})\|Z_i\|^{-2}[s^2+2ns+k^2\lambda_i^4]^{-1}$ and $\overline{f(s)}$, as in that case by using the theorem for convolution of originals is possible to write the relation:

$$A_i^*(t)=\int_0^t h_i(\tau)f(t-\tau)d\tau.$$

The original $h_i(t)=L^{-1}[\overline{H_i(s)}]$ was found from the third theorem of decomposition:

$$h_i(t)=\underset{s=s_1,s_2}{\mathrm{Res}}[e^{st}\overline{H_i(s)}]=\underset{s=s_1=-n-i\sqrt{k^2\lambda_i^4-n^2}_1}{\mathrm{Res}}[e^{st}\overline{H_i(s)}]+\underset{s=s_2=-n+i\sqrt{k^2\lambda_i^4-n^2}}{\mathrm{Res}}[e^{st}\overline{H_i(s)}]= \quad (15)$$
$$=Z_i(\xi_{sh})\|Z_i\|^{-2}(k^2\lambda_i^4-n^2)^{-1/2}e^{-nt}\sin(t\sqrt{k^2\lambda_i^4-n^2}).$$

Then for the solution of Eqn (12) is obtained the following:

$$A_i^*(t)=Z_i(\xi_{sh})\|Z_i\|^{-2}(k^2\lambda_i^4-n^2)^{-1/2}\int_0^t e^{-n\tau}\sin(\tau\sqrt{k^2\lambda_i^4-n^2})f(t-\tau)d\tau. \quad (16)$$

while the approximate solution of (9) acquires the form:

$$w^*(\xi,t)=\sum_{i=1}^{n^*}\frac{Z_i(\xi_{sh})\|Z_i\|^{-2}}{\sqrt{k^2\lambda_i^4-n^2}}Z_i(\xi)\int_0^t e^{-n\tau}\sin(\tau\sqrt{k^2\lambda_i^4-n^2})f(t-\tau)d\tau\equiv\int_0^t h(\xi,\tau)f(t-\tau)d\tau, \quad (17)$$
$$h(\xi,t)=e^{-nt}\sum_{i=1}^{n^*}\frac{Z_i(\xi_{sh})\|Z_i\|^{-2}}{\sqrt{k^2\lambda_i^4-n^2}}Z_i(\xi)\sin(t\sqrt{k^2\lambda_i^4-n^2}).$$

The initial conditions (3) allow to find the values of $\overline{A}_i,\alpha_i$: $\alpha_i=0,\ \overline{A}_i=0,\ \forall i>0\ \Rightarrow\ w_h(\xi,t)\equiv0$.

Finally must be mention that the solution of the non-homogeneous equation (1) is given by (17).

4. TRANSFER $H(\omega)$ AND IMPULSE $h(t)$ FUNCTIONS OF THE OSCILLATION PROCESS AT THE BEAM ENDPOINT

As the system has the ability linearity with constant in time characteristics, then there is a Linear Stationary System (LSS) which transforms the external force load $f(t)$ (input signal) into displacement of the beam endpoint (point A) - $w(t)$ (output signal): $w(t) = LSS\{f(t)\}$. The input can be presented as convolution:

$$f(t) = f * \delta(t) = \int_0^t f(\tau).\delta(t-\tau)d\tau ,$$

where $\delta(t)$ is an delta function. Since the convolution is linear in both of the multiplier, then for LSS the following is valid:

$$LSS\{f(t)\} \equiv LSS\{f * \delta(t)\} = f * LSS\{\delta(t)\}$$

After introducing the denotation $h(t) = LSS\{\delta(t)\}$ for the function giving the system response of a single impulse $\delta(t)$, the system operation is represented as a convolution between the input $f(t)$ and the impulse function $h(t)$:

$$w(t) = f * h(t) \equiv \int_0^t f(\tau).h(t-\tau)d\tau. \qquad (18)$$

The Fourier image $H(\omega)$ of the impulse function $h(t)$ is called transfer function. The image of output $w(t)$ as convolution represents a multiplication of the images of $h(t)$ and $f(t)$:

$$W(\omega) = H(\omega).Fce(\omega) \;\Rightarrow\; H(\omega) = W(\omega)/Fce(\omega).$$

On a Figs. 1 and 2 are given the force impulse, impulse and spectral functions of the system.

5. DETERMINATION OF THE OSCILLATION OF THE BEAM ENDPOINT FROM THE EXPERIMENTALLY MEASURED ITS ACCELERATION

It was experimentally measured the acceleration of the endpoint A of the beam after the applying of cross impact load, $\ddot{w}(t) \equiv acc(t)$. In Fig. 3 are presented the spectral function $Acc(\omega)$ and acceleration $acc(t)$. The spectral function of acceleration $Acc(\omega)$ is expressed by *Fast Fourier Transformation* (FFT)/ ω - circular frequency:

$$Acc(\omega) = \frac{1}{t_0}\int_0^{t_0} \ddot{w}(t)e^{-j.\omega t}dt , \; Acc(0) = \frac{\dot{w}(t_0) - \dot{w}(0)}{t_0}, \qquad (19)$$
$$A = FFT(a), \quad \dot{w}(t_0) = t_0 Acc(0) \; [\dot{w}(0) = 0],$$

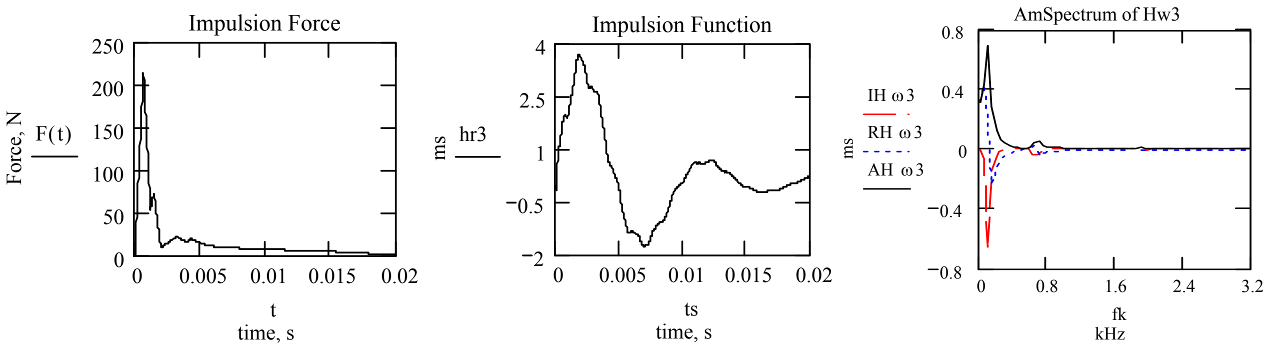

Fig. 1. Force impulse - $F(t)$, Impulse function – $hr3$ and Spectral function - $H\omega3$ for continuous model.

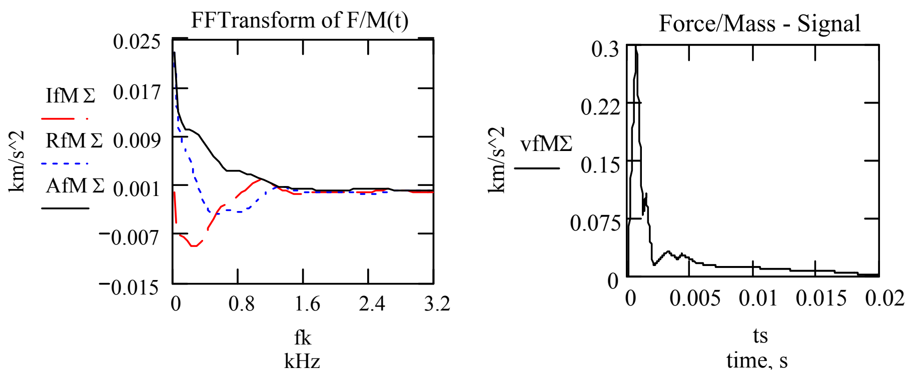

Fig. 2. Spectral function- $F(\omega)$ and force function.

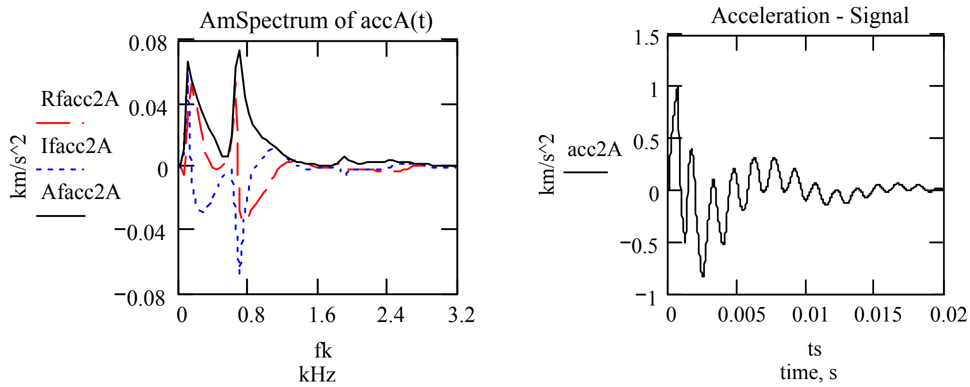

Fig. 3. Spectral function- *Acc(ω)* and acceleration.

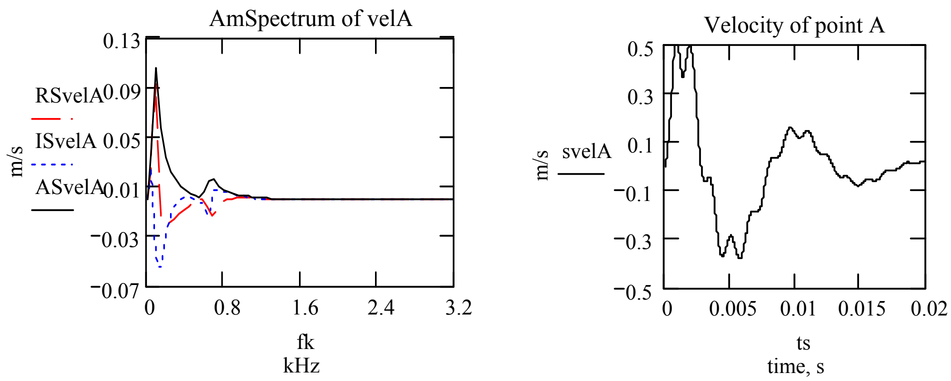

Fig. 4. Spectral function - $V(\omega)$ and original- $v(t)$.

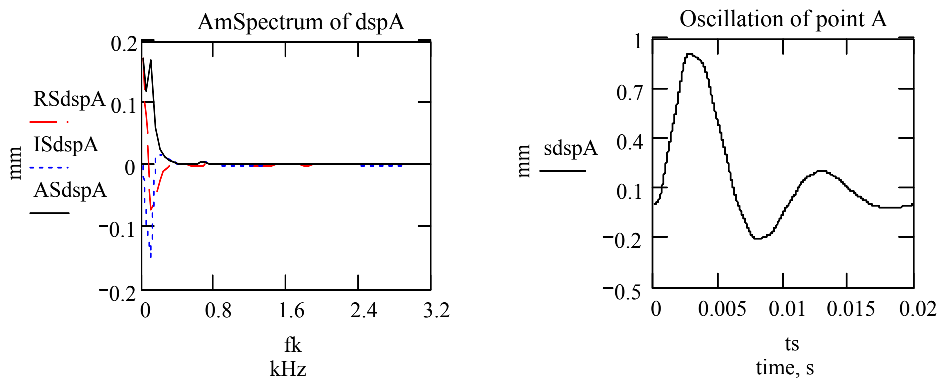

Fig.5. Spectral function- $W(\omega)$ and original- $w(t)$.

where t_0 is the duration of action of the impulse, $j=\sqrt{-1}$ - the imaginary unit, $a \equiv \{acc(t_i)\}_{i=0}^{N-1}$ - vector of discrete values of the acceleration with a power N, $A \equiv \{Acc(\omega_k)\}_{k=0}^{N_{fft}-1}$ - vector of discrete values of the spectral function with power N_{fft}, $\{t_i = i.\Delta t\}_{i=0}^{N-1}$, $\Delta t = t_0 / N;\ \omega_o = 2\pi / t_0, \{\omega_k = k.\omega_o\}_{k=0}^{N_{fft}-1}$ - discretization of time t and circular frequency ω. The spectral function of the velocity is determined by the relation:

$$Vel(\omega)=\frac{1}{t_0}\int_0^{t_0}\dot{w}(t)e^{-j.\omega.t}dt \equiv -\frac{1}{t_0.j.\omega}\int_0^{t_0}\dot{w}(t)de^{-j.\omega t} = \frac{1}{j\omega}[Acc(\omega)-\frac{\dot{w}(t_0)e^{-j.\omega.t_0}-\dot{w}(0)}{t_0}], \quad (20)$$

$$Vel(0)=\frac{w(t_0)-w(0)}{t_0} \Rightarrow w(t_0)=t_0.Vel(0)\ [w(0)=0].$$

After discretization we can find the vector:

$$V \equiv \{Vel(\omega_k)\}_{k=0}^{N_{fft}-1},\ Vel(\omega_k)=[Acc(\omega_k)-Acc(0)](j\omega_k)^{-1},\quad e^{-j.\omega_k.t_0}=1.$$

From the other hand the spectral function of displacement can be determined by the relation

$$Dsp(\omega)=\frac{1}{t_0}\int_0^{t_0}w(t)e^{-j.\omega t}dt \equiv -\frac{1}{t_0.j.\omega}\int_0^{t_0}w(t)de^{-j.\omega t} = \frac{1}{j.\omega}[Vel(\omega)-\frac{w(t_0)e^{-j.\omega t_0}-w(0)}{t_0}], \quad (21)$$

$$Dsp(0)=\frac{1}{t_0}\int_0^{t_0}w(t)dt,\quad e^{-j.\omega_k.t_0}=1.$$

Here after discretization the following vector is found:

$$W \equiv \{Dsp(\omega_k)\}_{k=0}^{N_{fft}-1},\ Dsp(\omega_k)=[Vel(\omega_k)-Vel(0)](j.\omega_k)^{-1}.$$

Meanwhile, since both $V_0 \equiv Vel(0)$ and $W_0 \equiv Dsp(0)$ are unknown, the new functions are introduced:

$$\bar{V}el(\omega) = Vel(0).\omega_o.\delta(\omega) + \tilde{V}el(\omega), \quad \tilde{V}el(\omega) = \begin{cases} Vel(\omega), |\omega| > 0.5\omega_o, \\ 0, \quad |\omega| \le 0.5\omega_o, \end{cases} \tag{22}$$
$$\bar{D}sp(\omega) = Dsp(0).\omega_o.\delta(\omega) + \tilde{D}sp(\omega), \quad \tilde{D}sp(\omega) = \begin{cases} Dsp(\omega), |\omega| > 0.5\omega_o, \\ 0, \quad |\omega| \le 0.5\omega_o, \end{cases}$$
$$\delta(\omega) = \begin{cases} \omega_o^{-1}, |\omega| > 0.5\omega_o, \\ 0, \quad |\omega| \le 0.5\omega_o. \end{cases}$$

In that case the *Inverse Fourier Transformation* (IFT) is applied to (22):

$$F^{-1}\{\bar{V}el(\omega)\} \equiv \bar{v}(t) = Vel(0) + \tilde{v}(t), \; \tilde{v}(t) \equiv F^{-1}\{\tilde{V}el(\omega)\}, \tag{23}$$
$$F^{-1}\{\bar{D}sp(\omega)\} \equiv \bar{w}(t) = Dsp(0) + \tilde{w}(t), \; \tilde{w}(t) \equiv F^{-1}\{\tilde{D}sp(\omega)\}.$$

The recovery of function becomes in terms of the average value at a given point:

$$\bar{v}(0) = [v(0_{-0}) + v(0_{+0})]/2 \equiv [\dot{w}(t_0) + \dot{w}(0)]/2 = Vel(0) + \tilde{v}(0), \tag{24}$$
$$\bar{w}(0) = [w(0_{-0}) + w(0_{+0})]/2 \equiv [w(t_0) + w(0)]/2 = Dsp(0) + \tilde{w}(0).$$

Now from (24) can be determining both unknowns:

$$Vel(0) = 0.5\dot{w}(t_0) - \tilde{v}(0), \quad Dsp(0) = 0.5w(t_0) - \tilde{w}(0). \tag{25}$$

By using the *Inverse Fast Fourier Transformation* (IFFT) both $\tilde{v}$ and $\tilde{w}$ are found:

$$\tilde{v} \equiv \{\tilde{v}_i\}_{i=0}^{N-1} = IFFT(\tilde{V}), \; \tilde{w} \equiv \{\tilde{w}_i\}_{i=0}^{N-1} = IFFT(\tilde{W}), \tag{26}$$
$$\tilde{V} = \{\tilde{V}el(\omega_k)\}_{k=0}^{N_{fft}-1}, \; \tilde{W} = \{\tilde{D}sp(\omega_k)\}_{k=0}^{N_{fft}-1}.$$

Finally the velocity and displacement are given by:

$$v = \{\dot{w}(t_i)\}_{i=0}^{N-1}, v_0 \equiv 0, \; v_i = Vel(0) + \tilde{v}_i, \; i = 1,\dots,N-1, \tag{27}$$
$$w = \{w(t_i)\}_{i=0}^{N-1}, w_0 \equiv 0, \; w_i = Dsp(0) + \tilde{w}_i, \; i = 1,\dots,N-1.$$

In Figs. 4 and 5 are given the spectral function of the velocity and the displacement of the point *A*.

6. TRANSFER FUNCTION AND RESPONSE OF THE SYSTEM IN THE ENDPOINT *A* BY THE EXPERIMENTALLY MEASURED ACCELERATION

The experimentally obtained spectral function of acceleration and its original are given on a Fig. 3. The spectral functions of the force - $Fce(\omega)$ and respectively of the displacement - $Dsp(\omega)$ are presented in Fig. 2 and Fig. 5.

The transfer function is a ratio of Fourier images of output to input: $H_{\exp}(\omega) = Dsp(\omega)/Fce(\omega)$. In discrete form the transfer function is:

$$H = \{H_{\exp}(\omega_k)\}_{k=0}^{N_{fft}-1}, \; F = \{Fce(\omega_k)\}_{k=0}^{N_{fft}-1}, \; H_{\exp}(\omega_k) = W_k / F_k.$$

The impulse function is sought through IFFT: $hr_{exp} = IFFT(H_{exp})$. The transfer function $H_{exp}(\omega)$ and system response $hr_{exp}(t)$ are given on Fig. 6.

Obviously, the type of the impulse function in Fig. 6 indicates the presence of instability associated with a critical loss of accuracy in the determination of the spectral function of the force at high values of the frequency, from where follows the same for the transfer function. It requires regularization of the solution related to the system response [11].

6.1 Regularization of the solution for system response

It is assumed that the impulse function $h(t), t \in [0, t_0]$ belongs to the Sobolev space $W^{(2)}$:

$$\|h\|_{W^{(2)}}^2 = \sum_{i=0}^{2} k_i \|h^{(i)}\|_2^2, \quad \|\bullet\|_2^2 = \frac{1}{t_0}\int_0^{t_0} |\bullet|^2 dt,$$

where $k_i, i = 0,1,2$ are the weighting coefficients.

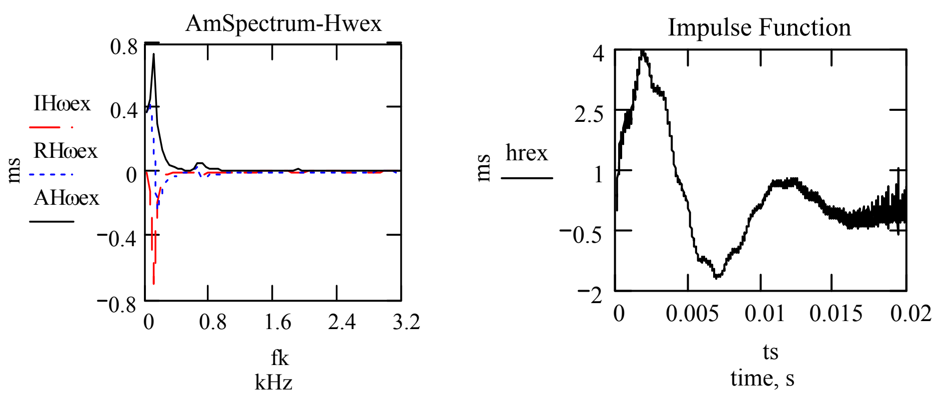

Fig. 6. Transfer and response functions at the endpoint.

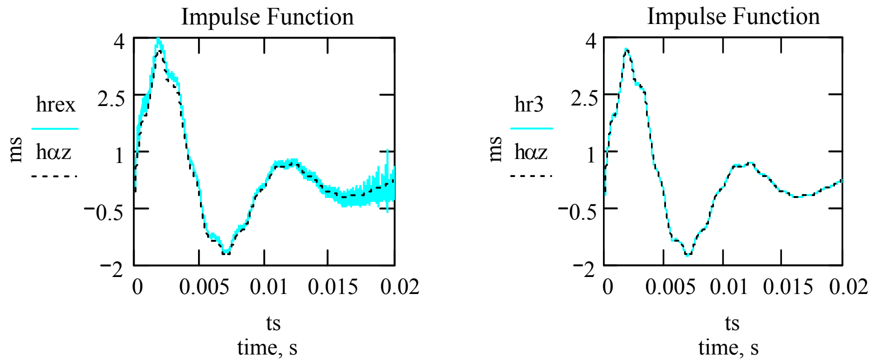

Fig. 7. Comparison of the impulse functions.

$W^{(2)}$ is the space of smooth square summable functions of second order. Let us now form the stabilizing functional:

$$\Phi(h_\alpha,\alpha)=\|hr_{ex}-h_\alpha\|_2^2+\alpha\sum_{i=0}^{2}k_i\|h_\alpha{}^{(i)}\|_2^2, \qquad (28)$$

where α is an regularization parameter. For determination of regularized solution must be found the minimum of the functional Φ:

$$h_\alpha:\ \min_h \Phi(h,\alpha). \qquad (29)$$

After discretization of the impulse function and its derivatives of first and second order:

$$\begin{aligned}&h=\{h_i\}_{i=0}^{N-1},\ h_i=h(t_i),\ t_i=i.\Delta t,\ \Delta t=t_0/N,\\ &h'_{i+1/2}=(h_{i+1}-h_i)\Delta t^{-1},\ h''_{i+1/2}=(h_i-2h_{i+1}+h_{i+2})\Delta t^{-2},\\ &i=0,1,\ldots,N-2;\ h_N\equiv h_0\end{aligned} \qquad (30)$$

can be determined the partial derivatives of the functional on the response function discrete values:

$$\begin{cases}\dfrac{\partial\Phi}{\partial h_0}=\dfrac{2}{N}(-b_0+A_0h_0+B_0h_1+C_0h_2)=0,\\ \dfrac{\partial\Phi}{\partial h_1}=\dfrac{2}{N}(-b_1+B_0h_0+A_1h_1+B_1h_2+C_1h_3)=0,\\ \dfrac{\partial\Phi}{\partial h_i}=\dfrac{2}{N}(-b_i+C_{i-2}h_{i-2}+B_{i-1}h_{i-1}+A_ih_i+B_ih_{i+1}+C_ih_{i+2})=0,\\ i=2,3,\ldots,N-3,\\ \dfrac{\partial\Phi}{\partial h_{N-2}}=\dfrac{2}{N}(-b_{N-2}+C_{N-4}h_{N-4}+B_{N-3}h_{N-3}+A_{N-2}h_{N-2}+B_{N-2}h_{N-1})=0,\\ \dfrac{\partial\Phi}{\partial h_{N-1}}=\dfrac{2}{N}(-b_{N-1}+C_{N-3}h_{N-3}+B_{N-2}h_{N-2}+A_{N-1}h_{N-1})=0.\end{cases} \qquad (31)$$

Here the vectors A,B,C and b are respectively:

$$\begin{aligned}&A=\{[0.5(\alpha+k_0)+\alpha.2.k_1.\Delta t^{-2}]d+\alpha.6k_2.\Delta t^{-4}.D\};\\ &B=0.25(\alpha+k_0)-\alpha.k_1.\Delta t^{-2}-\alpha.3k_2.\Delta t^{-4}.E,\ C=\alpha.3k_2.\Delta t^{-4}.\Lambda;\\ &b=\{b_i\}_{i=0}^{N-1},\ b_0=0.5(hr_{ex,0}+hr_{ex,1}),\ b_{N-1}=0.5(hr_{ex,N-2}+hr_{ex,N-1}),\\ &b_i=0.25(hr_{ex,i-1}+hr_{ex,i+1})+0.5hr_{ex,i},\ i=1,2,\ldots,N-2;\\ &d=\{d_i\}_{i=0}^{N-1},\ d_0=d_{N-1}=0.5,\ d_i=1,\ i=1,2,\ldots,N-2;\\ &D=\{D_i\}_{i=0}^{N-1},\ D_0=D_{N-1}=0.25,\ D_1=D_{N-2}=7/6,\ D_2=D_{N-3}=13/12,\\ &D_i=1,\ i=3,4,\ldots,N-4;\\ &E=\{E_i\}_{i=0}^{N-2},\ E_0=E_{N-1}=1,\ E_1=E_{N-2}=5/3,\ E_i=4/3,\ i=2,3,\ldots,N-3;\\ &\Lambda=\{\Lambda_i\}_{i=0}^{N-3},\ \Lambda_0=\Lambda_{N-3}=0.5,\ \Lambda_i=1/3,\ i=1,2,\ldots,N-4.\end{aligned}$$

The following system of linear equations by h_α is worked out:

$$P(\alpha).h_\alpha=b,\quad \alpha>0. \qquad (32)$$

Here the matrix $P(\alpha)$ is square of order N, symmetrical with five diagonals, which facilitates the search for a solution h_α by running forward and reverse. Family solutions $\{h_\alpha\}$ by α are found. Regularization parameter α^* is determined by the condition:

$$\alpha^*:\ \min_\alpha\|hr_{\exp}-h_\alpha\|_2^2. \qquad (33)$$

Finally for the regularized solution of the impulse function is found:

$$h_{\alpha*}=[P(\alpha^*)]^{-1}.b. \qquad (34)$$

In the Figure 7 the experimental $hr_{exp}(t)$ and theoretical $hr3(t)$ functions are compared with regularizated $h_{\alpha*}(t)$ impulse function.

6.2 Identification of the elasticity module of the PC structure

On the Fig. 6 is presented the spectral function $Acc(\omega)$ of the experimental measured acceleration in the beam endpoint A - $acc(t)$. The amplitude spectrum of acceleration gives three outstanding tops corresponding to quasi-natural linear frequencies $fr=\{fr_m\}_{m=0}^{2}\equiv(0.1,0.7,1.9)',\ kHz,$ while the first three eigenvalues of the boundary problem from (6) are $\lambda=(1.875,4.694,7.854)'$. The Young's modulus of PC structure is $E=13.55,\ GPa$. The coefficient, in which is involved in the Young's modulus of the material of the beam [10], can be obtained as the arithmetic mean of the ratios of the square of the natural angular frequencies of the system to the biquadrate of the corresponding eigenvalues of the boundary problem for the beam:

$$k^2 \equiv \frac{EI_y}{M_\Sigma . l_e^3} = \frac{1}{3}\sum_{m=0}^{2}\frac{(2.\pi.fr_m)^2 + n^2}{\lambda_m^4} \equiv k_{\exp}^2 = 37240,\ s^{-2}. \quad (35)$$

The Young's module and its relative change in the current case are respectively:

$$E_{\exp} = \frac{k_{\exp}^2 M_\Sigma l_e^3}{I_y} = 13.78\ ,\ GPa\ ,\ \psi_E = \left|1 - \frac{E_{\exp}}{E}\right| .100 = 1.685\%.$$

7. CONCLUSION

The main results from the current analysis can be concluded as follows:

- A non-homogeneous differential equation for transverse oscillations of console beam is derived.
- A theoretical solution for the transverse oscillations of a cantilever beam under transverse impact load is found.
- The velocity and displacement at the beam endpoint from experimentally measured acceleration are obtained.
- The transfer functions and response functions for the theoretical model by the experiment are defined.
- A regularized solution for impulse function obtained from experimental characteristic is found.
- The established model allows identification of the elasticity module of the beam material.
- It is created a mechanism for comparing the damping characteristics (coefficient of relative damping, logarithmic decrement of damping) of PC structures, which are used as construction materials.

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