Vol. 37, No. 2 (2015) 129-141

Tribology in Industry

www.tribology.fink.rs

RESEARCH

Jenkins Model Based Ferrofluid Lubrication of a Curved Rough Annular Squeeze Film with Slip Velocity

J.R. Patel[a], G.M. Deheri[a]

[a] *Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar, Anand, Gujarat, India.*

Keywords:

Annular plates
Roughness
Magnetic fluid
Jenkins model
Slip velocity

Corresponding author:

Jimit R. Patel
Department of Mathematics,
Sardar Patel University,
Vallabh Vidyanagar, Anand, Gujarat,
India-388120.
E-mail: patel.jimitphdmarch2013@gmail.com

A B S T R A C T

This paper deals with the combined effect of roughness and slip velocity on the performance of a Jenkins model based ferrofluid squeeze film in curved annular plates. Beavers and Joseph's slip model has been adopted to incorporate the effect of slip velocity. The stochastic model of Christensen and Tonder has been deployed to evaluate the effect of surface roughness. The associated stochastically averaged Reynolds type equation is solved to derive the pressure distribution, leading to the calculation of load carrying capacity. The graphical representation makes it clear that although, the effect of transverse surface roughness is adverse in general, Jenkins model based ferrofluid lubrication provides some measures in mitigating the adverse effect and this becomes more manifest when the slip parameter is reduced and negatively skewed roughness occurs. Of course, a judicious choice of curvature parameters and variance (-ve) add to this positive effect.

1. INTRODUCTION

Ferrofluid is a stable colloidal dispersion of magnetic nanoparticles. In the absence of magnetic fields, they behave as conventional Newtonian fluids. However, a magnetic body force appears under the presence of magnetic field gradients. This field-induced body force locally increases the magnetic pressure within the ferrofluid and promotes its migration towards the regions of positively field gradient. Interestingly, the viscosity of well-formatted ferrofluid does not change significantly with the superposition of a magnetic field.

Due to the wide application of the magnetic fluid, many authors have worked with magnetic fluids in different geometry of bearings. Tipei [1] worked on theory of lubrication using ferrofluids and applied it to short bearings. Agrawal [2] analyzed the performance of porous inclined slider bearing using a ferrofluid and found that the magnetization of magnetic particles in the lubricant increased the load carrying capacity. Sinha et al. [3] investigated the effect of ferrofluid lubrication on cylindrical rollers with cavitations. Ram and Verma [4] dealt with the performance of porous inclined slider bearing using ferrofluid lubrication.

Osman et al. [5] discussed the static and dynamic characteristics of magnetized journal bearings lubricated with ferrofluid. Shah and Bhat [6] studied the effect of ferrofluid lubrication on a squeeze film between curved annular plates considering rotation of magnetic particles. Deheri et al. [7] analyzed the performance of circular step bearings under the presence of a magnetic fluid. Ahmed and Singh [8] studied the effect of porous-pivoted slider bearing with slip velocity using ferrofluid. Urreta et al. [9] dealt with hydrodynamic bearing lubricated with magnetic fluids. Patel et al. [10] investigated the performance of a short hydrodynamic slider bearing in the presence of magnetic fluids. Patel et al. [11] discussed the performance of hydrodynamic short journal bearings lubricated with magnetic fluids. All the above investigations are established that the performance of the bearing system got enhanced due to magnetizations.

The squeeze film performance between various geometrical configurations is considered in a number of investigations (Archibald [12], Cameron [13], Prakash and Vij [14], Hamrock [15], Bhat [16]). Murti [17], Gupta and Vora [18], Ajwaliya [19], Bhat [16], Deheri and Abhangi [20] and Shimpi and Deheri [21] analyzed the effect of curvature parameters on the performance of squeeze film in rough plates.

Reduction of friction is relatively essential for the effective performance of a bearing system. It is found that slip velocity supports to reduce the friction. Beavers and Joseph [22] dealt with the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary condition at the interface. Flow with slip becomes useful for problems in chemical engineering for example, flows through pipes in which chemical reactions occur at the walls. Patel [23] studied the performance of hydro-magnetic squeeze film between porous circular disks with velocity slip. Many investigations have discussed theoretically and experimentally the effects of slip on various types of bearings (Thompson and Troian [24], Zhu and Granick [25], Spikes and Granick [26], Salant and Fortier [27], Wu et al. [28], Ahmed and Singh [8], Patel and Deheri [29], Wang et al. [30]). In all the above studies, it was manifest that the slip effect significantly affected the bearing system. Rao et al. [31] investigated the effects of velocity slip and viscosity variation in squeeze film lubrication of two circular plates. Recently, Patel and Deheri [32] investigated the combined effect of slip Velocity and roughness on magnetic fluid based infinitely long bearings.

In all the above investigations, smooth bearing surfaces were considered. But it is unrealistic because, the bearing surfaces develop roughness after having some run-in and wear. Many methods have been mooted to deal with the effect of surface roughness on the performance characteristics of squeeze film bearing. Christensen and Tonder [33-35] modified the stochastic theory of Tzeng and Saibel [36] to study the effect of surface roughness in general. A number of studies considered the stochastic model of Christensen and Tonder [33-35] to study the effect of surface roughness (Ting [37], Prakash and Tiwari [38], Guha [39], Gupta and Deheri [40], Nanduvinamani et al. [41], Deheri et al. [42], Chiang et al. [43], Bujurke et al. [44], Patel et al. [45], Shimpi and Deheri [46], Deheri et al. [47]). Shimpi and Deheri [48] analyzed the behaviour of a magnetic fluid based squeeze film between rotating transversely rough porous annular plates incorporating elastic deformation. Patel and Deheri [49] dealt with the effects of various porous structures on the performance of a Shliomis model based ferrofluid lubrication of a squeeze film in rotating rough porous curved circular plates. It was found that the adverse effect of transverse roughness could be overcome by the positive effect of ferrofluid lubrication in the case of negatively skewed roughness by suitably choosing curvature parameters and rotational inertia when Kozeny- Carman's model was deployed for porous structure. Patel and Deheri [50] theoretically studied the effect of Shliomis model based ferrofluid lubrication on the squeeze film between curved rough annular plates with comparison between two different porous structures. It was shown that the effect of morphology parameter and volume concentration parameter increased the load carrying capacity. Lastly, Patel and Deheri [51] discussed the effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough circular plates. It was established that for enhancing the performance characteristics of the bearing system the slip parameter was required to be reduced even if variance (−ve) occurs and suitable magnetic strength was in force.

The objective of the current study is to theoretically analyze the performance of Jenkins model based ferrofluid lubrication of a curved rough annular squeeze film with slip velocity.

2. ANALYSIS

The bearing configuration displayed in Fig. 1 consists of two annular plates each of inside radius b and outside radius a. The upper plate and lower plate are curved. Here r is the radial coordinates and h_0 is the central film thickness.

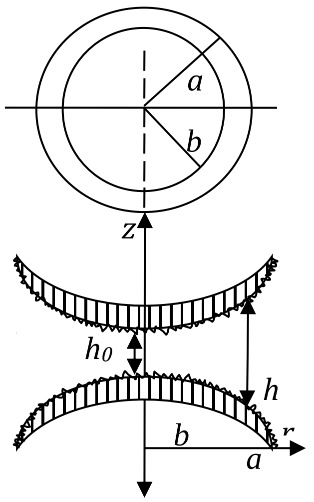

Fig. 1. Configuration of the bearing system.

The bearing surfaces are considered transversely rough. According to the stochastic theory of Christensen and Tonder [33-35], the thickness h of the lubricant film is considered as:

$$h = \bar{h} + h_s \quad (1)$$

where $\bar{h}$ represents the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is governed by the probability density function:

$$f(h_s) = \begin{cases} \frac{35}{32c^7}\left(c^2 - {h_s}^2\right)^3, -c \le h_s \le c \\ 0 \quad\quad\quad\quad , \; elsewhere \end{cases}.$$

wherein c is the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε which is the measure of symmetry of the random variable h_s, are considered according to the theory of Christensen and Tonder [33-35].

It is assumed that the upper plate lying along the surface determined by (Bhat [16], Abhangi and Deheri [52], Patel and Deheri [53]):

$$z_u = h_0 exp(-\beta r^2);\; b \le r \le a$$

approaches with normal velocity $\dot{h_0}$ to the lower plate lying along the surface governed by:

$$z_l = h_0[exp(-\gamma r^2) - 1];\; b \le r \le a$$

where β and γ denote the curvature parameter of the corresponding plates and h_0 is the central film thickness. The film thickness $h(r)$ then is defined by (Bhat [16], Abhangi and Deheri [52], Patel and Deheri [54]):

$$h(r) = h_0[exp(-\beta r^2) - exp(-\gamma r^2) + 1];$$
$$b \le r \le a$$

In 1972, a simple model to express the flow of a magnetic fluid was proposed by Jenkins. In this paper the magnetisable liquid was regarded as an anisotropic fluid and added to the motion and the temperature, the vector magnetization density to complete the description of the material. The use of local magnetization as an independent variable allowed Jenkins to treat static and dynamic situation in a uniform fashion and to make a natural distinction between paramagnetic and ferromagnetic fluids. A uniqueness theorem was recognized for incompressible paramagnetic fluids and determined that in these materials the magnetization vanished with the applied magnetic field. So the Jenkins model is not only a generalization of the Neuringer- Rosensweig model but also modifies both the pressure and the velocity of the magnetic fluid.

With Maugin's modification, equations of the model for steady flow are (Jenkins [55] and Ram and Verma [4]):

$$\rho(\bar{q}.\nabla)\bar{q} = -\nabla_p + \eta\nabla^2\bar{q} + \mu_0(\bar{M}.\nabla)\bar{H}$$
$$+\frac{\rho A^2}{2}\nabla \times \left[\frac{\bar{M}}{M} \times \{(\nabla \times \bar{q}) \times \bar{M}\}\right] \quad (2)$$

together with:

$$\nabla.\bar{q} = 0, \nabla \times \bar{H} = 0, \bar{M} = \bar{\mu}\bar{H}, \nabla.(\bar{H} + \bar{M}) = 0$$

(Bhat [16]). where ρ represents the fluid density, $\bar{q}$ denotes the fluid velocity in the film region, $\bar{H}$ is external magnetic field, $\bar{\mu}$ represents magnetic susceptibility of the magnetic fluid, p is the film pressure, η denotes the fluid viscosity, μ_0 is the permeability of the free space, A being a material constant and $\bar{M}$ is magnetization vector. From the above equation one concludes that Jenkins model is a generalization of Neuringer-Rosensweig model with an additional term:

$$\frac{\rho A^2}{2}\nabla\times\left[\frac{\bar{M}}{M}\times\{(\nabla\times\bar{q})\times\bar{M}\}\right]$$
$$=\frac{\rho A^2\bar{\mu}}{2}\nabla\times\left[\frac{\bar{H}}{H}\times\{(\nabla\times\bar{q})\times\bar{H}\}\right] \quad (3)$$

which modifies the velocity of the fluid. At this point one observes that Neuringer- Rosensweig model modifies the pressure while Jenkins model modifies both the pressure and velocity of the ferrofluid.

Let (u,v,w) be the velocity of the fluid at any point (r,θ,z) between two solid surfaces, with OZ as axis. Making the assumptions of hydrodynamic lubrication and remembering that the flow is steady and axially symmetric, the equations of motion are:

$$\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)\frac{\partial^2 u}{\partial z^2}=\frac{1}{\eta}\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right) \quad (4)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial w}{\partial z}=0 \quad (5)$$

Solving the above equation (4) under the boundary conditions, $u=0$ when $z=0,h$, one obtains:

$$u=\frac{z(z-h)}{2\eta\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right) \quad (6)$$

Substituting the value of u in equation (5) and integrating it with respect to z over the interval $(0,h)$ one gets Reynolds type equation for film pressure as:

$$\frac{1}{r}\frac{d}{dr}\left(\frac{h^3}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right)=12\eta\dot{h_0}$$

For the stochastic averaging theory of the differential equation, a method has been proposed by Christensen and Tonder [33-35]. Here an attempt has been made to modify this method, which on certain simplifications yields, under the usual assumptions of hydro-magnetic lubrication (Bhat [16], Prajapati [56], Deheri et al. [42]) the modified Reynolds type equation:

$$\frac{1}{r}\frac{d}{dr}\left(\frac{g(h)}{\left(1-\frac{\rho A^2\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_0\bar{\mu}}{2}H^2\right)\right)=12\eta\dot{h_0} \quad (7)$$

with:

$$g(h)=\begin{pmatrix}h^3+3h^2\alpha+3(\sigma^2+\alpha^2)h\\+3\sigma^2\alpha+\alpha^3+\varepsilon\end{pmatrix}\left(\frac{4+sh}{2+sh}\right)$$

and

$$H^2=K(r-b)(a-r)$$

Where K is a suitably chosen constant so as to have a magnetic field of required strength, which suits the dimensions.

Following dimensionless quantities are introduced:

$$\bar{h}=\frac{h}{h_0}=[exp(-BR^2)-exp(-CR^2)+1],$$
$$R=\frac{r}{b},P=-\frac{h_0^3p}{\eta b^2\dot{h_0}},B=\beta b^2,C=\gamma b^2,$$
$$\mu^*=-\frac{K\mu_0\bar{\mu}h_0^3}{\eta\dot{h_0}},k=\frac{a}{b},\bar{A}^2=\frac{\rho A^2\bar{\mu}b\sqrt{K}}{2\eta},$$
$$\bar{\sigma}=\frac{\sigma}{h_0},\bar{\alpha}=\frac{\alpha}{h_0},\bar{\varepsilon}=\frac{\varepsilon}{h_0^3},\bar{s}=sh_0 \quad (8)$$

Using the equation (8), equation (7) transforms to:

$$\frac{1}{R}\frac{d}{dR}\left(\frac{g(\bar{h})}{\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)}R\frac{d}{dR}\left(P-\frac{1}{2}\mu^*(R-1)(k-R)\right)\right)=-12 \quad (9)$$

where:

$$g(\bar{h})=\begin{pmatrix}\bar{h}^3+3\bar{h}^2\bar{\alpha}+3(\bar{\sigma}^2+\bar{\alpha}^2)\bar{h}\\+3\bar{\sigma}^2\bar{\alpha}+\bar{\alpha}^3+\bar{\varepsilon}\end{pmatrix}\left(\frac{4+\bar{s}\bar{h}}{2+\bar{s}\bar{h}}\right)$$

Solving equation (9) under the boundary conditions:

$$P(1)=P(k)=0 \quad (10)$$

one derives the expression for the dimensionless pressure distribution as:

$$P=\frac{\mu^*}{2}(R-1)(k-R)$$
$$-6\int_1^R\frac{R}{g(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR$$

$$+6\frac{\int_1^k \frac{R}{g(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR}{\int_1^k \frac{1}{Rg(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR}$$

$$\int_1^R \frac{1}{Rg(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR \qquad (11)$$

The dimensionless load carrying capacity of the bearing system then, is determined by:

$$W=-\frac{h_0^3 w}{2\pi\eta b^4 \dot{h_0}}=\frac{\mu^*}{24}(k+1)(k-1)^3$$

$$+3\int_1^k \frac{R^3}{g(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR$$

$$-3\frac{\left[\int_1^k \frac{R}{g(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR\right]^2}{\int_1^k \frac{1}{Rg(\bar{h})}\left(1-\bar{A}^2\sqrt{(R-1)(k-R)}\right)dR} \qquad (12)$$

3. RESULTS AND DISCUSSION

It is clearly seen that the non-dimensional pressure increases by:

$$\frac{\mu^*}{2}(R-1)(k-R)$$

while the increase in the load carrying capacity is found to be:

$$\frac{\mu^*}{24}(k+1)(k-1)^3$$

as compared to the case of conventional lubricant based bearing system. This is because the viscosity of the lubricant gets increased due to magnetization. Furthermore, as the expression involved in equation (12) is linear with respect to magnetization, an increase in the magnetization parameter could result in increased load carrying capacity.

The rate of increase in the load carrying capacity due to magnetization is presented in Figs. 2-9. It is clearly observed that load carrying capacity increases sharply due to magnetization.

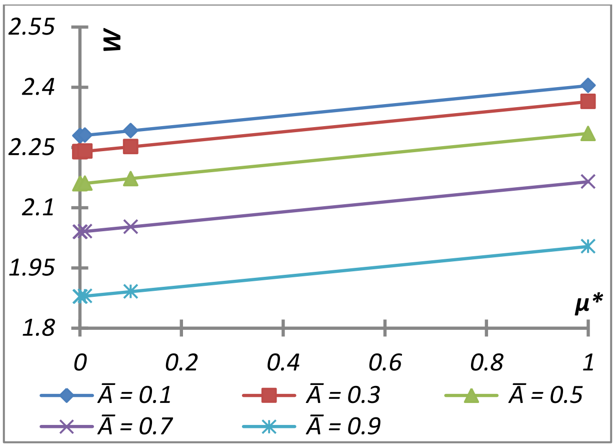

Fig. 2. Variation of Load carrying capacity with respect to μ^* and $\bar{A}$.

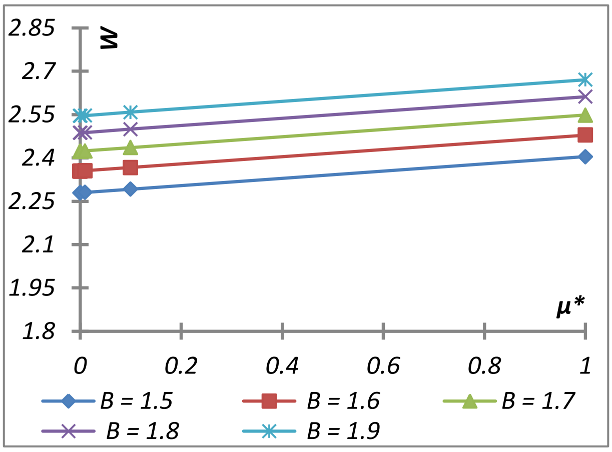

Fig. 3. Variation of Load carrying capacity with respect to μ^* and B.

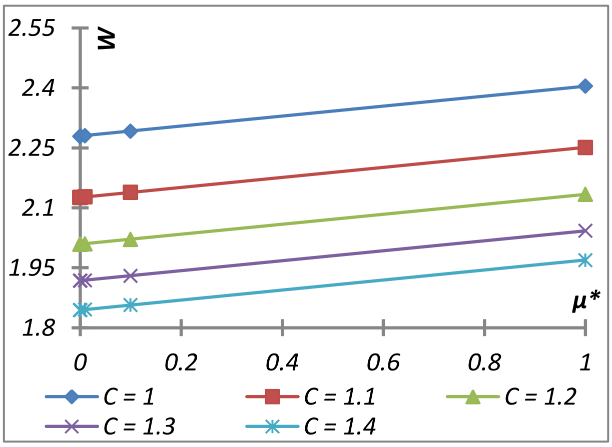

Fig. 4. Variation of Load carrying capacity with respect to μ^* and C.

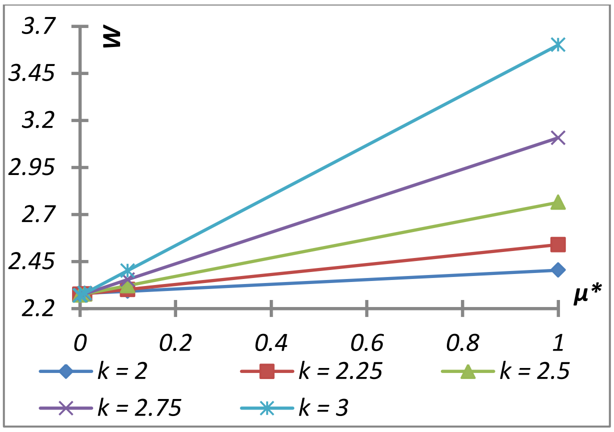

Fig. 5. Variation of Load carrying capacity with respect to μ^* and k.

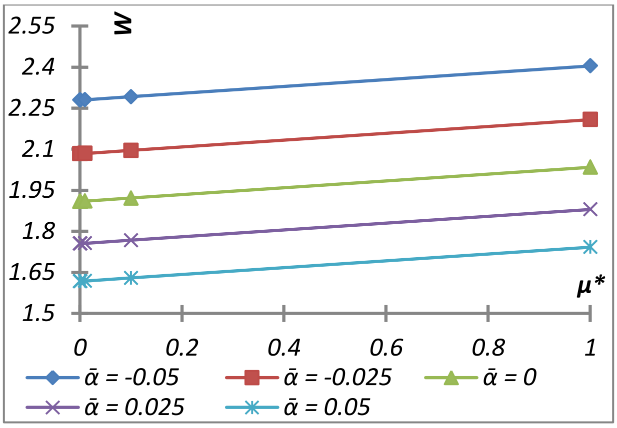

Fig. 8. Variation of Load carrying capacity with respect to μ^* and $\bar{\alpha}$.

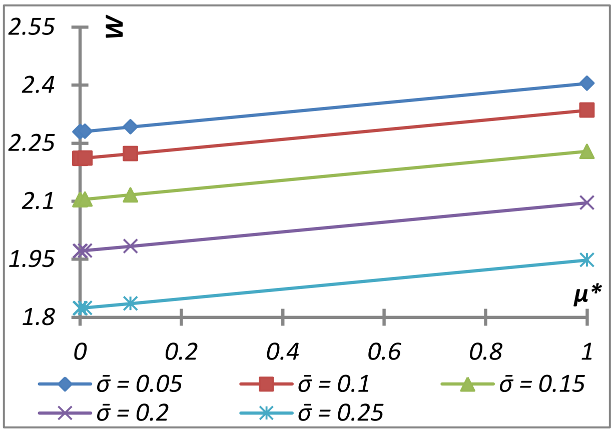

Fig. 6. Variation of Load carrying capacity with respect to μ^* and $\bar{\sigma}$.

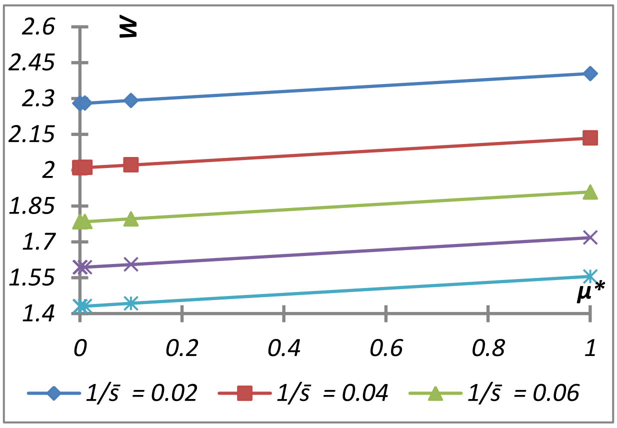

Fig. 9. Variation of Load carrying capacity with respect to μ^* and $1/\bar{s}$.

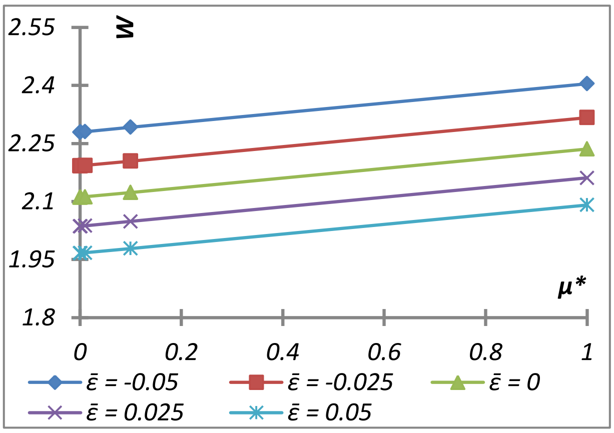

Fig. 7. Variation of Load carrying capacity with respect to μ^* and $\bar{\varepsilon}$.

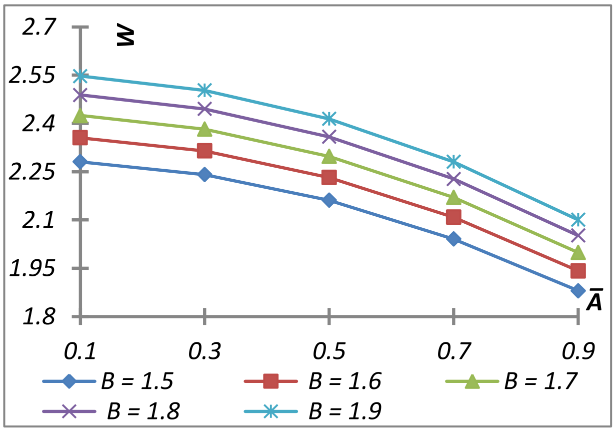

Fig. 10. Variation of Load carrying capacity with respect to $\bar{A}$ and B.

The fact that the material constant parameter has an adverse effect is displayed in Figs. 10-16. The combined effect of aspect ratio and material constant is to reduce the load carrying capacity substantially, as can be seen from Fig. 12.

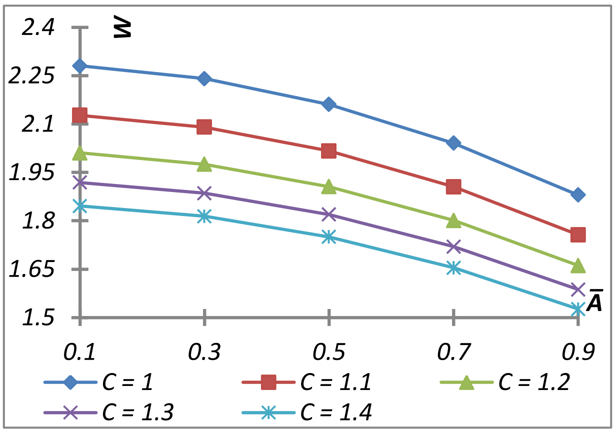

Fig. 11. Variation of Load carrying capacity with respect to $\bar{A}$ and C.

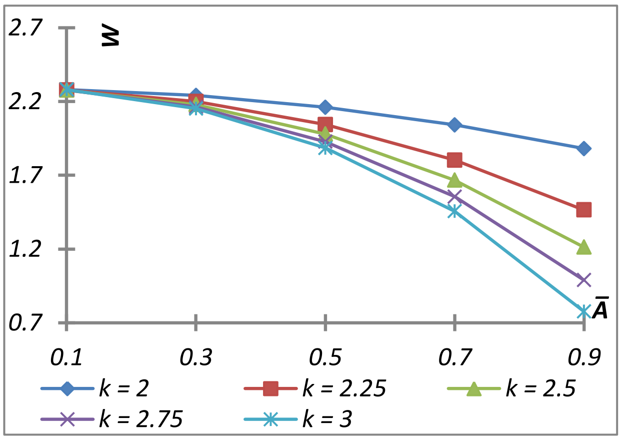

Fig. 12. Variation of Load carrying capacity with respect to $\bar{A}$ and k.

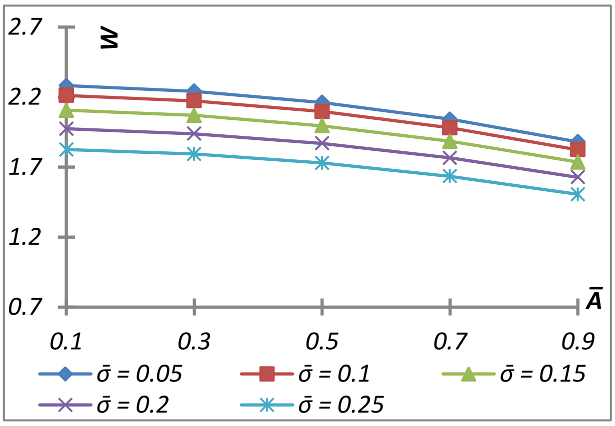

Fig. 13. Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\sigma}$.

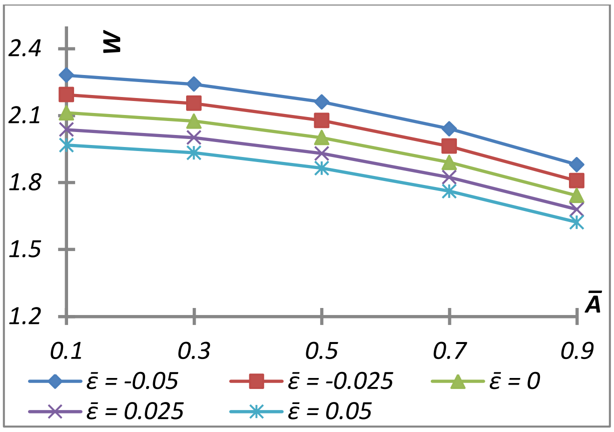

Fig. 14. Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\varepsilon}$.

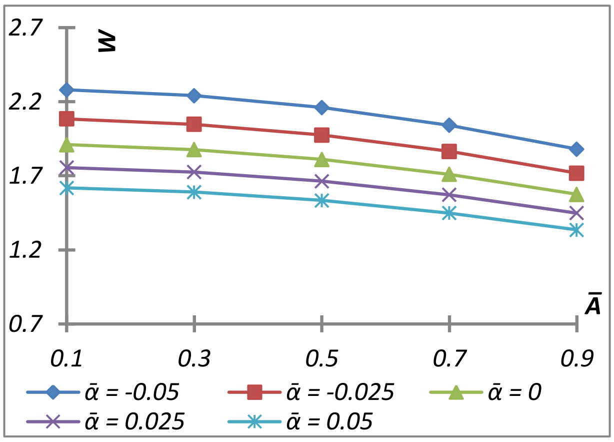

Fig. 15. Variation of Load carrying capacity with respect to $\bar{A}$ and $\bar{\alpha}$.

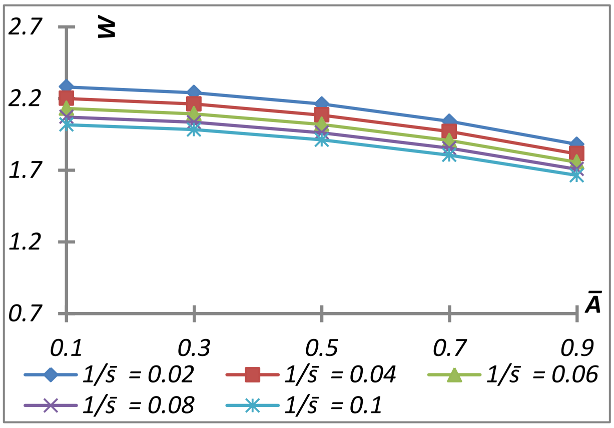

Fig. 16. Variation of Load carrying capacity with respect to $\bar{A}$ and $1/\bar{s}$.

It is observed from Figs. 17-22 that the load carrying capacity increases sharply with increase in upper plate's curvature parameter.

However, the effect of aspect ratio on the distribution of load carrying capacity with respect to upper plate's curvature parameter remains nominal, for higher values of aspect ratio.

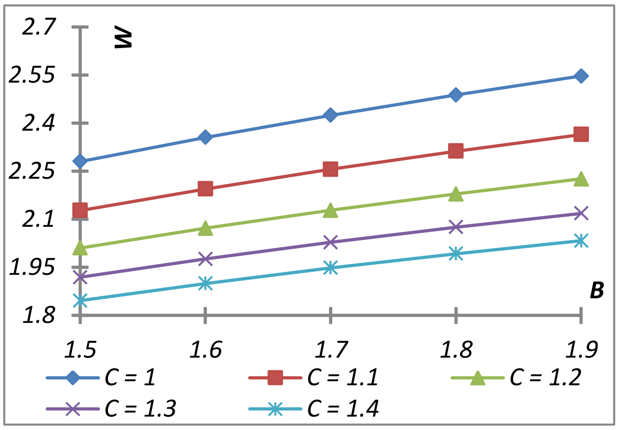

Fig. 17. Variation of Load carrying capacity with respect to B and C.

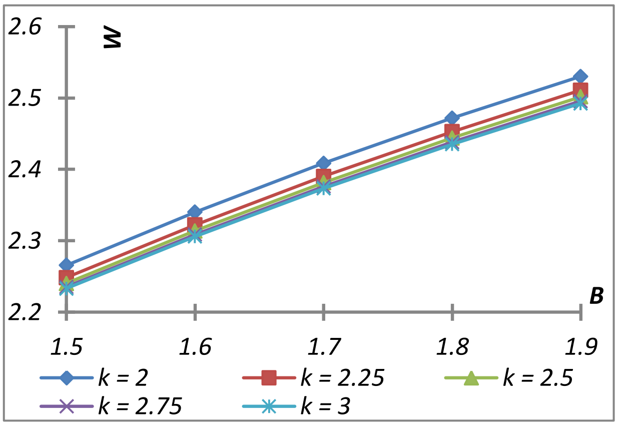

Fig. 18. Variation of Load carrying capacity with respect to B and k.

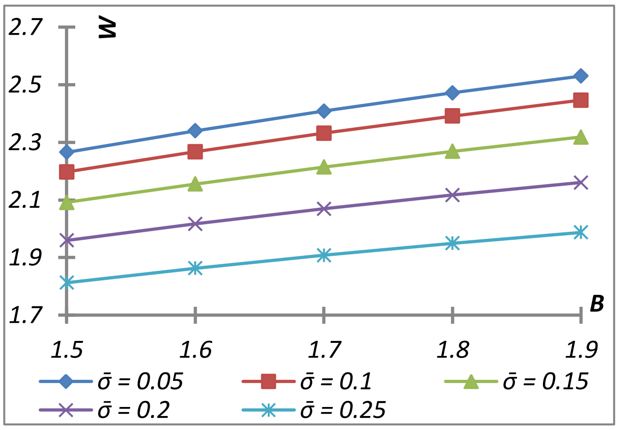

Fig. 19. Variation of Load carrying capacity with respect to B and $\bar{\sigma}$.

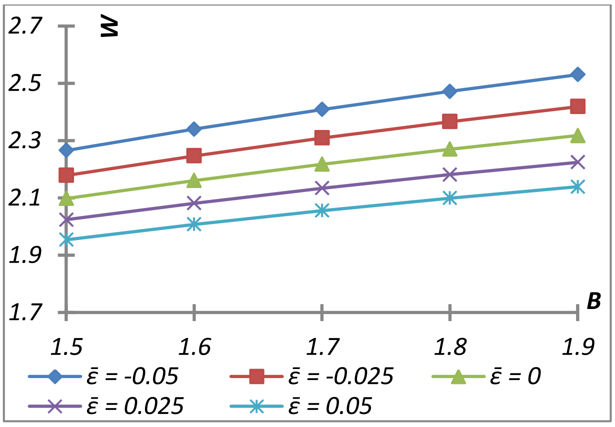

Fig. 20. Variation of Load carrying capacity with respect to B and $\bar{\varepsilon}$.

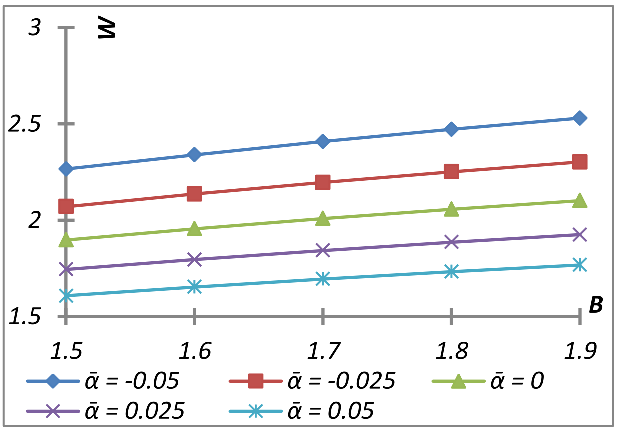

Fig. 21. Variation of Load carrying capacity with respect to B and $\bar{\alpha}$.

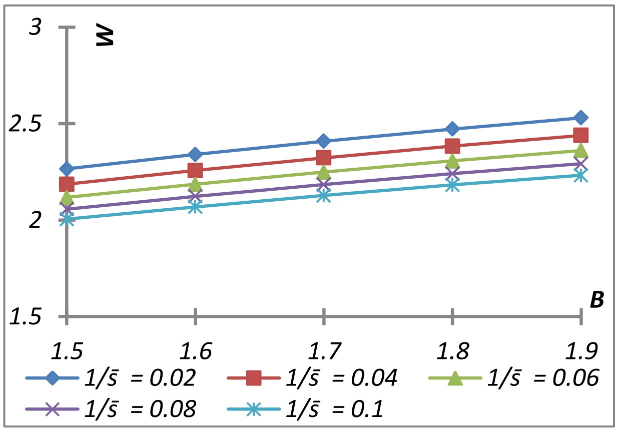

Fig. 22. Variation of Load carrying capacity with respect to B and $1/\bar{s}$.

It appears that the trends of load carrying capacity with respect to lower plate's curvature parameters are opposite to that of the upper plate's curvature parameter. This can be observed from Figs. 23-27. Therefore, for improving the performance of bearing system the ratio of curvature parameters must be considered judiciously.

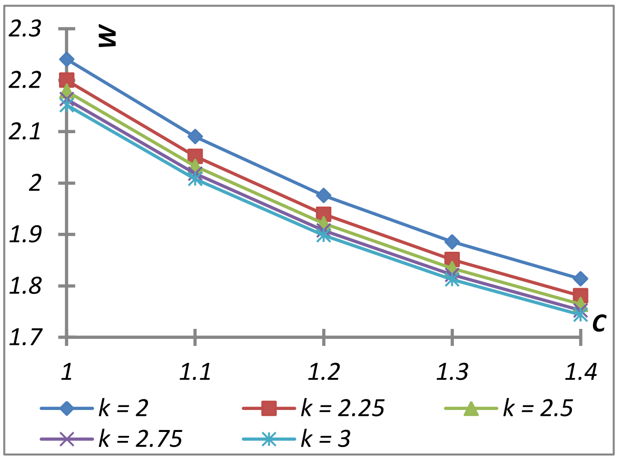

Fig. 23. Variation of Load carrying capacity with respect to C and k.

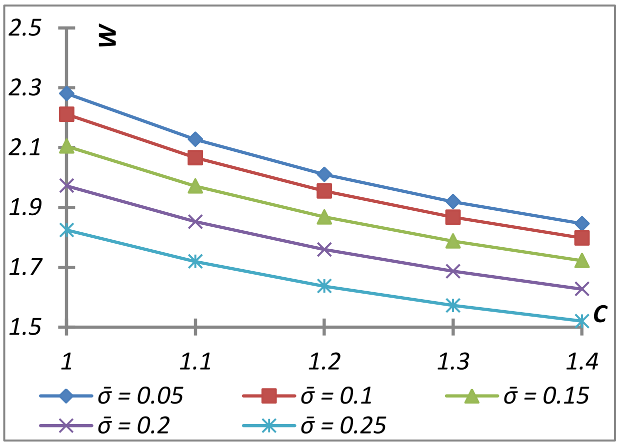

Fig. 24. Variation of Load carrying capacity with respect to C and $\bar{\sigma}$.

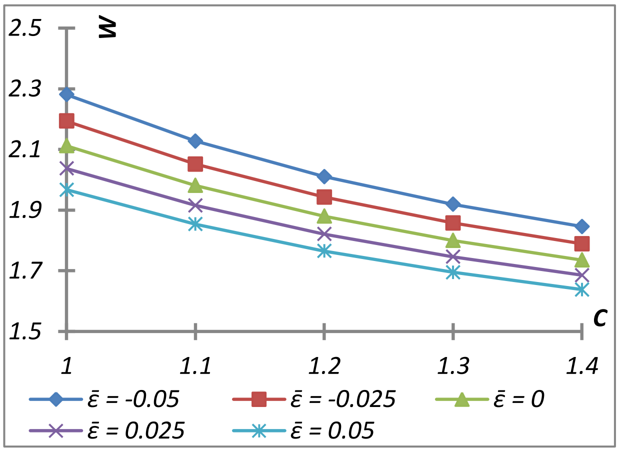

Fig. 25. Variation of Load carrying capacity with respect to C and $\bar{\varepsilon}$.

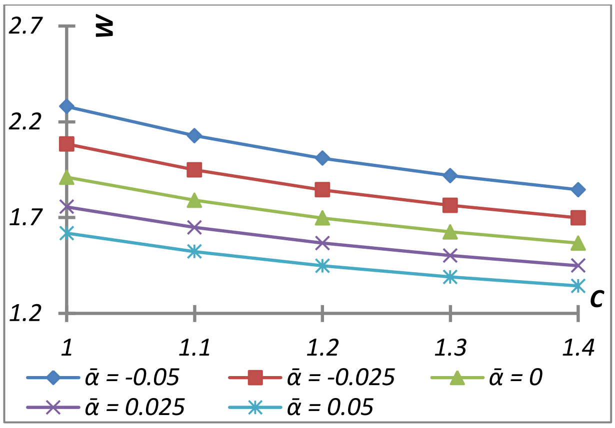

Fig. 26. Variation of Load carrying capacity with respect to C and $\bar{\alpha}$.

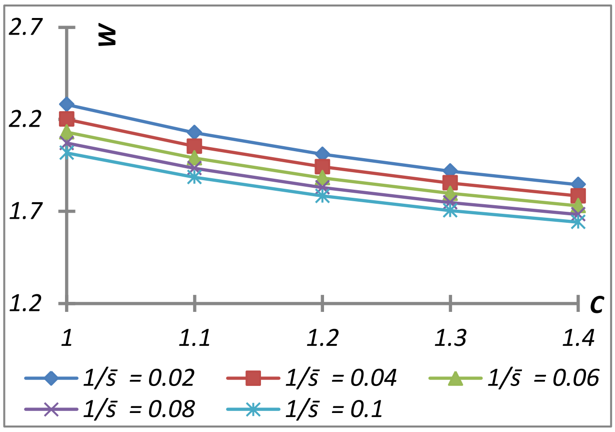

Fig. 27. Variation of Load carrying capacity with respect to C and $1/\bar{s}$.

Figures 28-31 suggest that the load carrying capacity decreases with the increase in aspect ratio. Therefore, the role of aspect ratio is central to the enhanced performance of the bearing system.

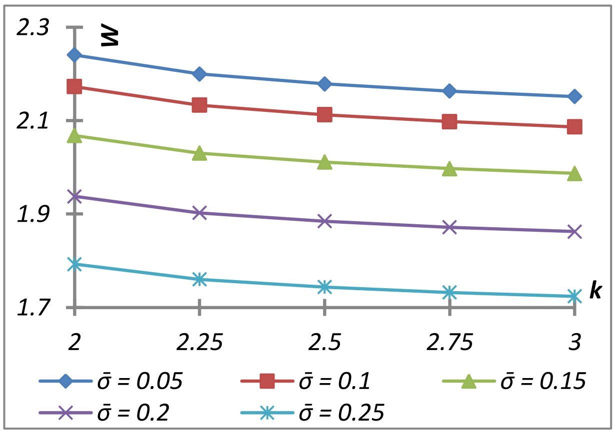

Fig. 28. Variation of Load carrying capacity with respect to k and $\bar{\sigma}$.

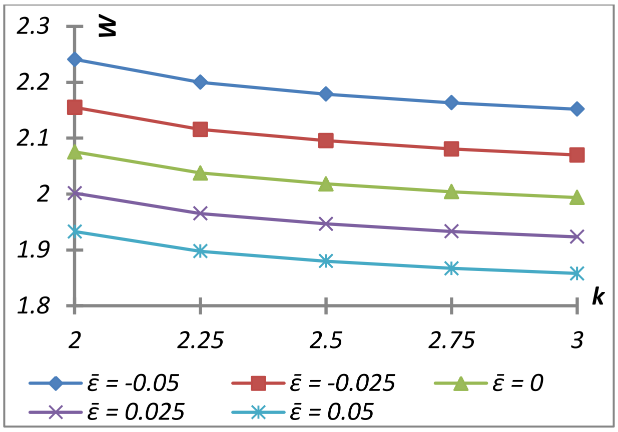

Fig. 29. Variation of Load carrying capacity with respect to k and $\bar{\varepsilon}$.

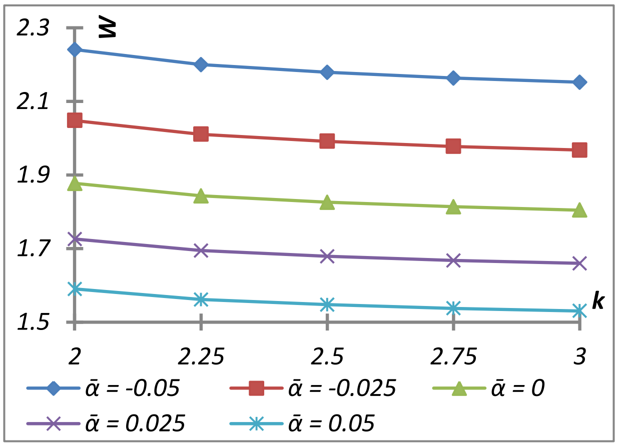

Fig. 30. Variation of Load carrying capacity with respect to k and $\bar{\alpha}$.

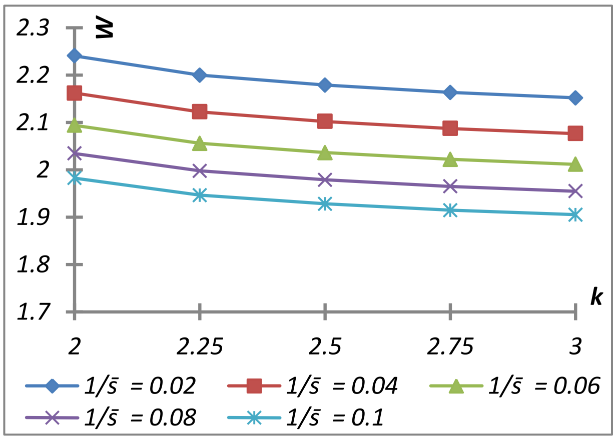

Fig. 31. Variation of Load carrying capacity with respect to k and $1/\bar{s}$.

The standard deviation associated with roughness adversely affects the performance of the bearing system, which is given in Figs. 32-34. This is not surprising as the motion of the lubricant gets retarded owing to roughness. Figure 34 underlines that the combined effect of slip velocity and standard deviation is to bring down the load carrying capacity.

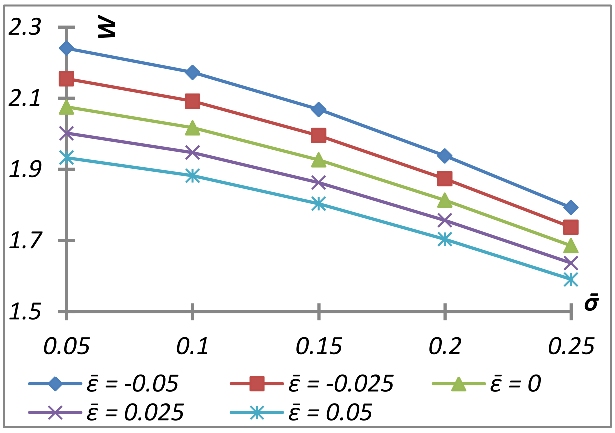

Fig. 32. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\varepsilon}$.

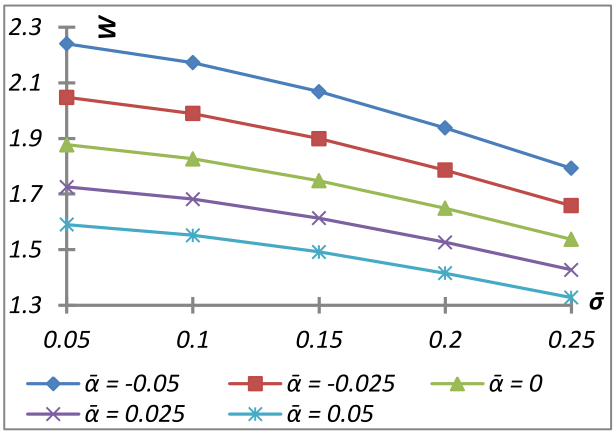

Fig. 33. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$.

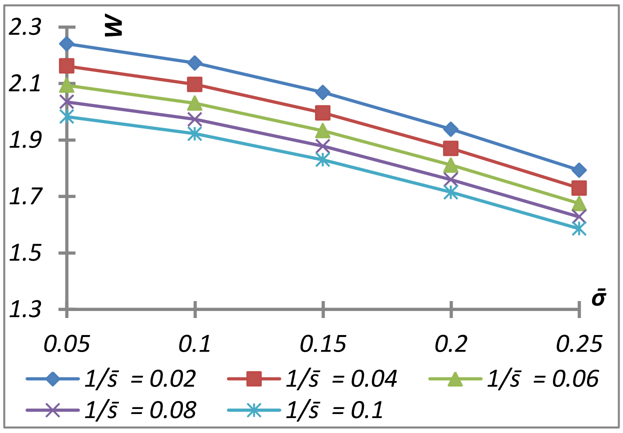

Fig. 34. Variation of Load carrying capacity with respect to $\bar{\sigma}$ and $1/\bar{s}$.

The increased load carrying capacity due to variance (-ve) gets further increased in the case of negatively skewed roughness, which is clear from Fig. 35. Lastly, Figs. 36 and 37 indicate that in any case the slip parameter deserves to be minimized if at all the performance is to be enhanced.

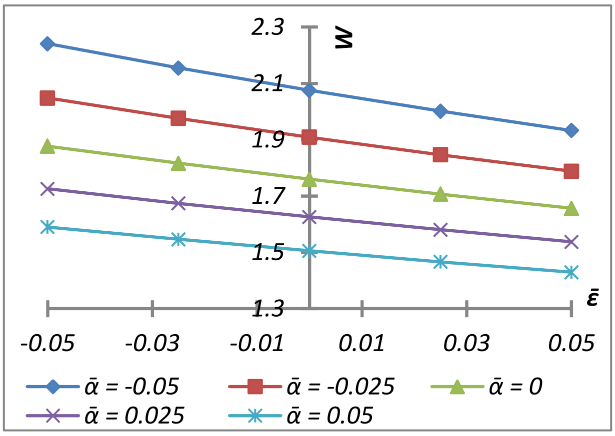

Fig. 35. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $\bar{\alpha}$.

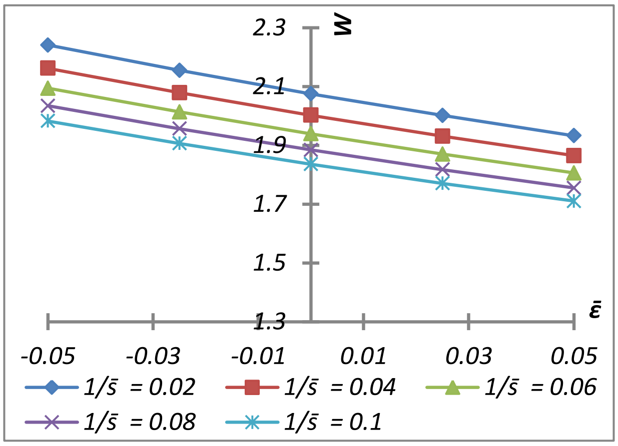

Fig. 36. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and $1/\bar{s}$.

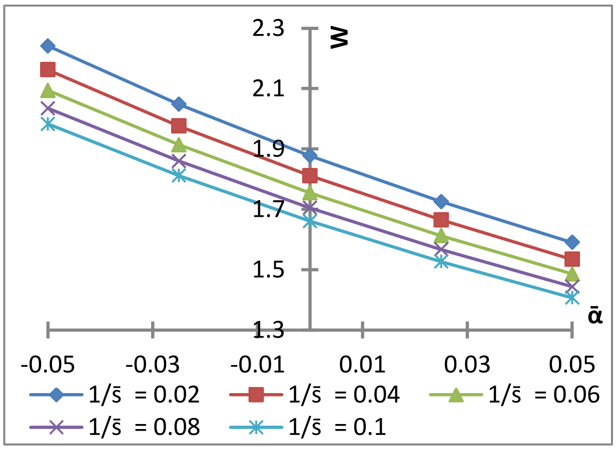

Fig. 37. Variation of Load carrying capacity with respect to $\bar{\alpha}$ and $1/\bar{s}$.

4. CONCLUSION

It is clearly observed that Jenkins model scores over the Neuringer-Rosensweig model in improving the performance of the bearing system. Even if suitable magnetic strength is in place, the roughness aspect must be considered carefully while designing the bearing system all though slip parameter is reduced. This study offers the suggestion that the adverse effect of standard deviation and slip velocity can be compensated up to a large extent by the Jenkins model based magnetic fluid flow, at least in the case of negatively skewed roughness. Besides, it is revealed that this type of bearing system supports certain amount of load, even when there is no flow, contrary to the case of conventional lubricant based bearing system.

Acknowledgement

Jimit R. Patel acknowledges the funding given by the UGC in the priority program UGC-BSR Research Fellowship. The authors acknowledge with thanks the fruitful comments and suggestions of the reviewers and editor.

REFERENCES

[1] N. Tipei, 'Theory of Lubrication with Ferrofluids: Application to short bearings', *Transactions of ASME*, vol. 104, no. 4, pp. 510-515, 1982.

[2] V.K. Agrawal, 'Magnetic fluid based porous inclined slider bearing', *Wear*, vol. 107, no. 2, pp. 133-139, 1986.

[3] P. Sinha, P. Chandra and D. Kumar, 'Ferrofluid lubrication of cylindrical rollers with cavitation', *Acta Mechanica*, vol. 98, no. 1-4, pp. 27-38, 1993.

[4] P. Ram and P.D.S. Verma, 'Ferrofliuid lubrication in porous inclined slider bearing', *Indian Journal of Pure and Applied Mathematics*, vol. 30, no. 12, pp. 1273-1281, 1999.

[5] T.A. Osman, G.S. Nada and Z.S. Safar, 'Static and dynamic characteristics of magnetized journal bearings lubricated with ferrofluid', *Tribology International*, vol. 34, no. 6, pp. 369–380, 2001.

[6] R.C. Shah and M.V. Bhat, 'Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles', *Journal of Engineering Mathematics*, vol. 51, no. 4, pp. 317-324, 2005.

[7] G.M. Deheri, H.C. Patel and R.M. Patel, 'Performance of magnetic fluid based circular step bearings', *MECHANIKA*, vol.57, no.1, pp. 22-27, 2006.

[8] N. Ahmad and J.P. Singh, 'Magnetic Fluid lubrication of porous-pivoted slider bearing with slip velocity', *Journal of Engineering Tribology*, vol. 221, no. 5, pp. 609-613, 2007.

[9] H. Urreta, Z. Leicht, A. Sanchez, A. Agirre, P. Kuzhir and G. Magnac, 'Hydrodynamic bearing lubricated with magnetic fluids', *Journal of Physics: Conference Series*, vol. 149, no. 1, 2009.

[10] R.M. Patel, G.M. Deheri and P.A. Vadher, 'Performance of a Magnetic Fluid-based Short Bearing', *Acta Polytechnica Hungarica*, vol. 7, no. 3, pp. 63-78, 2010.

[11] N.S. Patel, D.P. Vakharia and G.M. Deheri, 'A Study on the Performance of a Magnetic-Fluid-Based Hydrodynamic Short Journal Bearing', *ISRN Mechanical Engineering*, vol. 2012, pp. 1-7, 2012.

[12] F.R. Archibald, 'Load capacity and time relations for squeeze films', *Trans. ASME*, vol. 78, pp. A231– A245, 1956.

[13] A. Cameron, *The Principles of Lubrication,* Longmans, London, 1966.

[14] J. Prakash and S.K. Vij, 'Load capacity and time height relation between porous plates', *Wear*, vol. 24, pp. 309–322, 1973.

[15] B.J. Hamrock, *Fundamentals of Fluid Film Lubrication,* McGraw-Hill, New York, 1994.

[16] M.V. Bhat, *Lubrication with a Magnetic fluid*, India: Team Spirit (India) Pvt. Ltd, India, 2003.

[17] P.R.K. Murti, 'Squeeze films in curved circular plates', *Trans. ASME*, vol. 97, no. 4, pp. 650–652, 1975.

[18] J.L. Gupta and K.H. Vora, 'Analysis of squeeze film between curved annular plates', *J. Lubr. Technol. Trans. ASME*, vol. 102, pp. 48–59, 1980.

[19] M.B. Ajwaliya, *On certain theoretical aspects of lubrication,* Dissertation, Sardar Patel University Vallabh Vidyanagar, India, 1984.

[20] G.M. Deheri and N.D. Abhangi, 'Squeeze film based on magnetic fluid in curved rough circular plates', *Journal of Engineering annals of Faculty of engineering Hunedoara*, vol. VI, no. 2, pp. 95-106, 2008.

[21] M.E. Shimpi and G.M. Deheri, 'Ferrofluid Lubrication of Rotating Curved Rough Porous Circular Plates and Effect of Bearing's Deformation', *Arabian Journal for Science and Engineering*, vol. 38, no. 10, pp. 2865-2874, 2013.

[22] G.S. Beavers, D.D. Joseph, 'Boundary conditions at a naturally permeable wall', *Journal of Fluid Mechanics*, vol. 30, pp. 197-207, 1967.

[23] K.C. Patel, 'The hydromagnetic squeeze film between porous circular disks with velocity slip', *Wear*, vol. 58, no. 2, pp. 275-281, 1980.

[24] P.A. Thompson and S.M. Troian, 'A general boundary condition for liquid flow at solid surfaces', *Nature*, vol. 389, pp. 360-362, 1997.

[25] Y. Zhu and S. Granick, 'Rate-Dependent Slip of Newtonian Liquid at Smooth Surfaces', *Physical review letters*, vol. 87, no. 9, pp. 096105-1-4, 2001.

[26] H. Spikes and S. Granick, 'Equation for Slip of Simple Liquids at Smooth Solid Surfaces', *Langmuir*, vol. 19, no. 12, pp. 5065-5071, 2003.

[27] R.F. Salant and A.E. Fortier, 'Numerical Analysis of a Slider Bearing with a Heterogeneous Slip/No-Slip Surface', *Tribology Transactions*, vol. 47, no. 3, pp. 328-334, 2004.

[28] C.W. Wu, G.J. Ma, P.Zhou and C.D. Wu, 'Low Friction and High Load Support Capacity of Slider Bearing With a Mixed Slip Surface', *Journal of Tribology*, vol. 128, no. 4, pp. 904-907, 2006.

[29] N.D. Patel and G.M. Deheri, 'Effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing with slip velocity', *J. Serb. Soc. Comput. Mech.*, vol. 5, no. 1, pp. 104-118, 2011.

[30] L.L. Wang, C.H. Lu, M. Wang and W.X. Fu, The Numerical Analysis of the Radial Sleeve Bearing With Combined Surface Slip, *Tribology International*, vol. 47, pp. 100–104, 2012.

[31] R.R. Rao, K. Gouthami and J.V. Kumar, 'Effect of velocity-slip and viscosity variation in squeeze film lubrication of two circular plates', *Tribology in Industry*, vol. 35, no. 1, pp. 51-60, 2013.

[32] J.R. Patel and G. Deheri, 'Slip Velocity and Roughness Effect on Magnetic Fluid Based Infinitely Long Bearings', in: *Proceedings of International Conference on Advances in Tribology and Engineering Systems Lecture Notes in Mechanical Engineering, Springer*, Gujarat, India, pp. 97-109, 2014.

[33] H. Christensen and K.C. Tonder, 'Tribology of rough surfaces: stochastic models of hydrodynamic lubrication', *SINTEF*, Report no. 10, pp. 69-18, 1969a.

[34] H. Christensen and K.C. Tonder, 'Tribology of rough surfaces: parametric study and comparison of lubrication models', *SINTEF*, Report no. 22, pp. 69-18, 1969b.

[35] H. Christensen and K.C. Tonder, 'The hydrodynamic lubrication of rough bearing surfaces of finite width', in: *ASME-ASLE Lubrication Conference. Cincinnati. OH*., Paper no. 70-lub-7, October 12-15, 1970.

[36] S.T. Tzeng and E. Saibel, 'Surface roughness effect on slider bearing lubrication', *ASLE Trans.*, vol. 10, no. 3, pp.334-348, 1967.

[37] L. Ting: 'Engagement behavior of lubricated porous annular disks. Part I: Squeeze film phase -- surface roughness and elastic deformation effects', *Wear*, vol. 34, no. 2, pp. 159-172, 1975.

[38] J. Prakash and K. Tiwari, 'Lubrication of a Porous Bearing with Surface Corrugations', *Jour. of Tribo.*, vol. 104, no. 1, pp. 127-134, 1982.

[39] S. Guha, 'Analysis of dynamic characteristics of hydrodynamic journal bearings with isotropic roughness effects', *Wear*, vol. 167, no. 2, pp. 173-179, 1993.

[40] J.L. Gupta and G.M. Deheri, 'Effect of roughness on the behavior of squeeze film in a spherical bearing', *Tribol Trans.*, vol. 39, no. 1, pp. 99-102, 1996.

[41] N.B. Naduvinamani, S.T. Fathima and P.S. Hiremath, 'Hydrodynamic lubrication rough Slider bearings with couple stress fluids', *Tribology International*, vol. 36, no. 12, pp. 949-959, 2003.

[42] G.M. Deheri, P.I. Andharia and R.M. Patel, 'Longitudinally rough slider bearings with Squeeze film formed by a magnetic fluid', *Industrial Lubrication and Tribology*, vol. 56, no. 3, pp. 177-187, 2004.

[43] H.L. Chiang, C.H. Hsu and J.R. Lin, 'Lubrication performance of finite journal bearings considering effects of couple stresses and surface roughness', *Tribology International*, vol. 37, no. 4, pp. 297–307, 2004.

[44] N.M. Bujurke, N.B. Naduvinamani and D.P. Basti, 'Effect of surface roughness on the squeeze film lubrication between curved annular plates', *Industrial Lubrication and Tribology*, vol. 59, no. 4, pp.178–185, 2007.

[45] H.C. Patel, G.M. Deheri, R.M. Patel, 'Magnetic fluid-based squeeze film between porous rotating rough circular plates', *Industrial Lubrication and Tribology*, vol. 61, no. 3, pp.140-145, 2009.

[46] M.E. Shimpi and G.M. Deheri, 'Surface roughness and elastic deformation effects on the behavior of the magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets', *Tribology in Industry*, vol. 32, no. 2, pp. 21-30, 2010.

[47] G.M. Deheri, R.M. Patel and N.D. Abhangi, 'Magnetic fluid-based squeeze film behavior between transversely rough curved annular plates: a comparative study', *Industrial Lubrication and Tribology*, vol. 63, no. 4, pp. 254–270, 2011.

[48] M.E. Shimpi and G.M. Deheri, 'Magnetic Fluid-Based Squeeze Film Behaviour in Curved Porous-Rotating Rough Annular Plates and Elastic Deformation Effect', *Advances in Tribology*, vol. 2012, 2012.

[49] J.R. Patel and G. Deheri, 'Shliomis Model Based Ferrofluid Lubrication of Squeeze Film in Rotating Rough Curved Circular Disks with Assorted Porous Structures', *American Journal of Industrial Engineering,* vol. 1, no. 3, pp. 51-61, 2013.

[50] J.R. Patel and G. Deheri, 'Theoretical study of Shliomis model based magnetic squeeze film in rough curved annular plates with assorted porous structures', *FME Transactions*, vol. 42, no. 1, pp. 56-66, 2014.

[51] J.R. Patel and G. Deheri, 'Combined Effect of Surface Roughness and Slip Velocity on Jenkins Model Based Magnetic Squeeze Film in Curved Rough Circular Plates', *International Journal of Computational Mathematics,* vol. 2014, pp. 1-9, 2014.

[52] N.D. Abhangi and G.M. Deheri, 'Numerical modeling of squeeze film performance between rotating transversely rough curved circular plates under the presence of a magnetic fluid lubricant', *ISRN Mechanical Engineering*, vol. 2012, pp. 1-9, 2012.

[53] J.R. Patel and G. Deheri, 'Shliomis model based magnetic fluid lubrication of a squeeze film in rotating rough curved circular plates, Caribbean' *Journal of Science and Technology*, vol. 1, pp. 138-150, 2013.

[54] J.R. Patel and G. Deheri, 'Shliomis model-based magnetic squeeze film in rotating rough curved circular plates: a comparison of two different porous structures', *Int. J. of Computational Materials Science and Surface Engineering*, vol. 6, no. 1, pp. 29 – 49, 2014.

[55] J.T. Jenkins, 'A theory of magnetic fluids', *Arch. Rational Mech. Anal.*, vol. 46, no. 1, pp. 42-60, 1972.

[56] B. L. Prajapati, 'On Certain Theoretical Studies in Hydrodynamic and Electro-magneto hydrodynamic Lubrication', *Ph. D. Thesis, S.P. University, Vallabh Vidya- Nagar*, 1995.